

Investment, Time, and Present Value

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Introduction

Decisions made by a firm manager today often result in cash flows which occur not only today but also years into the future. For example, a manufacturer may purchase a machine today—an immediate cost—which results in higher revenues for the firm for years to come, since the machine enhances the firm's production process.

It is a bit tricky to evaluate the merits of a decision whose costs and benefits occur over time. For example, suppose I offered you an investment option—invest \$100 and receive a return of \$200. Is this a good investment? The answer depends when you get the return of \$200. If you get it next year, then it's good. If you get it in 75 years then it ain't so good.

So how does one precisely measure costs and benefits of actions which have financial consequences over time? The concept of present value is very useful. We shall discuss present value presently. But first, let's look at a closely related topic that tends to be more intuitive—*future value*.

Future value

Suppose you are offered a certain financial investment—place \$100 in the investment today, and receive a 5% annual rate of return (simple interest¹), for as long as you want.

What will your investment's value be in 1 year?

$$\text{Future value of \$100 in 1 year, 5\% annual return} = 100 \times 1.05 = \$105$$

How about 2 years? (In the second year, the \$105 will earn 5% interest.)

$$\text{Future value of \$100 in 2 years, 5\% annual return} = 105 \times 1.05 = \$110.25$$

How about 3 years? (In the third year, the \$110.25 will earn 5% interest.)

$$\text{Future value of \$100 in 3 years, 5\% annual return} = 110.25 \times 1.05 = \$115.7625$$

¹ Throughout our analysis we shall assume simple interest—the equivalent of compounding interest annually. In corporate finance classes, you will encounter other types of interest, e.g. compounded monthly, continuously, etc. We also ignore taxes.

Oh, if only there were a formula that could calculate the future value of a sum of money. Hey, there is such a formula! It's on the next page of notes!

$$FV = PV(1 + i)^n$$

“FV” is future value

“PV” is present value—the value of the money today

i is the rate of return (like an interest rate), in its decimal form

n is the number of years in the future

For example, how much will your \$100 investment be worth in 21 years?

$$FV = 100(1 + .05)^{21} = \$278.5963$$

(There are more complicated versions of this formula, for investments with uncertain rates of return, and ones with compounding occurring more than once a year, etc. You will encounter these more complicated formulas in corporate finance classes.)

We have seen how to calculate the future value of \$ invested today. But can we calculate the value today—the present value—of a future sum? Yes!

Present Value

Suppose some dude offered you a certain investment—he will certainly give you \$1000 ten years from now. How much are you willing to pay him now to receive a certain \$1000 in ten years? You will pay less than \$1000, since you can invest the money yourself in a Treasury bond that pays 6% rate of return (and let's assume that this is a certain payment).

We can use the future value equation to calculate the maximum amount of money that you would be willing to pay the dude today to get \$1000 with certainty in the future:

$$FV = PV(1 + i)^n$$

Divide both sides by $(1 + i)^n$

$$\frac{FV}{(1 + i)^n} = PV$$

Now, for convention's sake, switch sides:

$$PV = \frac{FV}{(1 + i)^n} \quad \leftarrow \text{Hey! This is the present value equation!!!!}$$

Note that we divide FV by a number greater than 1, that depends on i – the rate of return. Hence PV is less than FV. (Hopefully this makes sense to you.) We are said to *discount* the future value; hence i is called the *discount rate*.

Let's use the present value equation to calculate the most that you would pay the dude today for his investment. Assume that you could receive a 6% rate of return with certainty elsewhere in the economy from other investments.

$$PV = \frac{1000}{(1 + .06)^{10}} = \$558.3948$$

The most that you'd be willing to pay NOW to get \$1000 in 10 years is \$558.3948. If you can pay less than that today and get \$1000 in 10 years then the investment is better than the Treasury bond option (at a 6% return). If you must pay more, than the Treasury bond option is superior.

The present value of a stream of cash flows

Suppose that you won the lotto, paying you \$1,000,000 per year for 25 years. Now some dudette offers to buy your winning lotto ticket for \$15 million. Should you accept the offer? (And let's ignore taxes here, as we have done throughout these notes.)

Well, we can use the present value formula 25 times, to calculate the present value of each of the \$1,000,000 payments. Then we can add up the 25 results, to get the present value of this stream of twenty-five \$1 million cash flows. (We should accept the dudette's \$15 million offer if it is greater than the present value of the alternative—the 25 payments.)

A difficulty arises: what do we use for i – the discount rate? We should use the rate of return that we would expect if we accept the dudette's buyout offer and invest the \$15 million in some financial instrument. This return is uncertain and is crucial in the calculation. For now, let's just assume an 8% return.

For example:

$$\text{PV of the first \$1 million} \quad PV = \frac{1000}{(1 + .08)^0} = \$1 \text{ million}$$

$$\text{PV of the second \$1 million} \quad PV = \frac{1000}{(1 + .08)^1} = \$925,926$$

In the handy table on the next page, I present the PVs of all the 25 payments, at 8% discount rate.

year	payment	pv of payment
0	1,000,000.00	1,000,000.00
1	1,000,000.00	925,925.93
2	1,000,000.00	857,338.82
3	1,000,000.00	793,832.24
4	1,000,000.00	735,029.85
5	1,000,000.00	680,583.20
6	1,000,000.00	630,169.63
7	1,000,000.00	583,490.40
8	1,000,000.00	540,268.88
9	1,000,000.00	500,248.97
10	1,000,000.00	463,193.49
11	1,000,000.00	428,882.86
12	1,000,000.00	397,113.76
13	1,000,000.00	367,697.92
14	1,000,000.00	340,461.04
15	1,000,000.00	315,241.70
16	1,000,000.00	291,890.47
17	1,000,000.00	270,268.95
18	1,000,000.00	250,249.03
19	1,000,000.00	231,712.06
20	1,000,000.00	214,548.21
21	1,000,000.00	198,655.75
22	1,000,000.00	183,940.51
23	1,000,000.00	170,315.28
24	1,000,000.00	157,699.34
		11,528,758.28

The dudette's offer of \$15 million looks awfully good, compared to the PV of the 25 payments—\$11.5 million! (In other words, if you take the dudette's offer of \$15 million and you earn an 8% return with it then you will do much better than keeping the lotto ticket—around \$3.5 million better in today's dollars.)

(By the way, some financial calculators can calculate the present value of a stream of equal cash flows using only a few commands. Consult your calculator's manual.)

Using Present Value to Analyze Investment Decisions: Net Present Value

Now suppose that you are a firm thinking of making an investment; it will cost money today, and will result in higher revenue in the future. There will be a cash outflow today, and cash inflows in the future.

You would like to evaluate the merits of this investment, relative to alternate investments that you could make. The *net present value* of the investment is the present value of all of the cash outflows and inflows that result from the investment.

If an investment's net present value is positive, then it is a superior investment relative to the average investment in the economy. The higher an investment's net present value, the more superior it is relative to the average investment.

Example:

A machine costs \$1000 to buy. It will result in \$300 of revenue at the end of each year, for 4 years. At the end of the four years it can be sold for scrap for \$100.

This investment has five cash flows:

Now:	-\$1000 (the cost of the machine)
1 year from now:	\$300
2 years from now:	\$300
3 years from now:	\$300
4 years from now:	\$300 + \$100 = \$400 (revenue + scrap value)

Let us use the present value formula to calculate the PV of each of these cash flows. Then we'll add up the PVs to get the Net Present Value (NPV).

One important question: what discount rate to use? Answer: the discount rate should equal the rate of return that an average investment with similar risk would enjoy. For now, let's just say that it is 11%.

$$\text{PV of cash flows} = \frac{-1000}{1.11^0} + \frac{300}{1.11^1} + \frac{300}{1.11^2} + \frac{300}{1.11^3} + \frac{400}{1.11^4} = -\$3.39$$

The negative NPV tells us that we could do better with an average investment elsewhere in the economy, so this one doesn't look very good.