MGMT 5113 Unit 9, Video 1

Hypothesis testing for a single population variance

Example: A laser machine tool is supposed to cut watch gears in precise thickness averaging 500 microns with standard deviation of 4 microns. You take a random sample of 10 watch gears and they have these thicknesses in microns:

500 490 510 501 499 502 497 503 500 499

Use the sample to test the claim that the tool cuts gears with a thickness variance of 4 microns. (Use a significance level of .10)

First, recall that we must use hypothesis testing on variances, not on standard deviations.

Recall:

Variance = standard deviation squared

Standard deviation = square root of variance

In our example, we have a claim of a population standard deviation of 4 microns. We must convert the standard deviation into a variance so that we can test it.

Variance = standard deviation^2 = 4^2 = 16 microns

Next, construct the null hypothesis (conventional wisdom):

Null hypothesis: Variance = 16

Alternate hypothesis: Variance ≠ 16
("variance is not equal to 16")

Because our alternate hypothesis does <u>not</u> have "<" or ">", we must do a <u>two-tailed test</u>.

Because we are using a two-tailed test, we must split the level of significance so that half of it is in each tail.

So each tail should have .05 in it.

When we do a hypothesis test for one population variance, we use a chi-square distribution, which is conveniently represented in our textbook by a chi-square table in the appendix (p. 946).

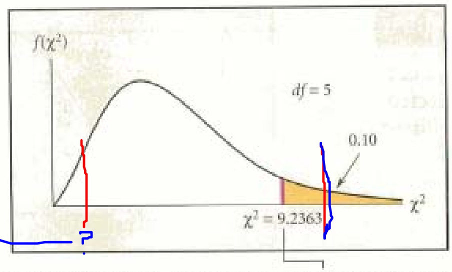
Degrees of freedom (df) = sample size - 1

For our example, degrees of freedom = 10 - 1 = 9

Our two critical values are **3.3251 and 16.9190**

APPENDIX G

Values of χ^2 for Selected Probabilities



.95 of the distribution To the right

PROBABILITIES (OR AREAS UNDER CHI-SQUARE DISTRIBUTION CURVE ABOVE GIVEN CHI-SQUARE VALUES)

df	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
	Values of Chi-Squared									
1	0.0000	0.0002	0.0010	0.0039	0.0158	2.7055	3.8415	5.0239	6.6349	7.8794
2	0.0100	0.0201	0.0506	0.1026	0.2107	4.6052	5.9915	7.3778	9.2104	10.5965
3	0.0717	0.1148	0.2158	0.3518	0.5844	6.2514	7.8147	9.3484	11.3449	12.8381
4	0.2070	0.2971	0.4844	0.7107	1.0636	7.7794	9.4877	11.1433	13.2767	14.8602
5	0.4118	0.5543	0.8312	1.1455	1.6103	9.2363	11.0705	12.8325	15.0863	16.7496
6	0.6757	0.8721	1.2373	1.6354	2.2041	10.6446	12.5916	14.4494	16.8119	18.5475
7	0.9893	1.2390	1.6899	2.1673	2.8331	12.0170	14.0671	16.0128	18.4753	20.2777
8	1.3444	1.6465	2.1797	2.7326	3.4895	13.3616	15.5073	17.5345	20.0902	21.9549
9	1.7349	2.0879	2.7004	3.3251	4.1682	14.6837	16.9190	19.0228	21.6660	23.5893
10	2.1558	2.5582	3.2470	3.9403	4.8652	15.9872	18.3070	20.4832	23.2093	25.1881
11	2.6032	3.0535	3.8157	4.5748	5.5778	17.2750	19.6752	21.9200	24.7250	26.7569
12	3.0738	3.5706	4.4038	5.2260	6.3038	18.5493	21.0261	23.3367	26.2170	28.2997
13	3.5650	4.1069	5.0087	5.8919	7.0415	19.8119	22.3620	24.7356	27.6882	29.8193
14	4.0747	4.6604	5.6287	6.5706	7.7895	21.0641	23.6848	26.1189	29.1412	31.3194
15	4.6009	5.2294	6.2621	7.2609	8.5468	22.3071	24.9958	27.4884	30.5780	32.8015

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Now, calculate the sample variance. Use this equation:

$$s^2 = \frac{\Sigma(x-\overline{x})^2}{n-1}$$

"s^2" is the sample variance.

The "x" with the bar on top is the sample mean. So first we need to calculate the sample mean:

500 +490+510+501+499+502+497+503+500+499 = 5,001

5001 / 10 = 500.1 <--the sample mean

Now, subtract each gear's thickness from the sample mean thickness:

500 - 500.1 = -0.1
490 - 500.1 = -10.1
510 - 500.1 = 9.9
501 - 500.1 = 0.9
499 - 500.1 = -1.1
502 - 500.1 = 1.9
497 - 500.1 = -3.1
503 - 500.1 = 2.9
500 - 500.1 = -0.1
499 - 500.1 = -1.1

Now, square each result:

-0.1 x -0.1 = 0.01
-10.1 x -10.1 = 102.01
9.9 x 9.9 = 98.01
0.9 x 0.9 = 0.81
-1.1 x -1.1 = 1.21
1.9 x 1.9 = 3.61
-3.1 x -3.1 = 9.61
2.9 x 2.9 = 8.41
-0.1 x -0.1 = 0.01
-1.1 x -1.1 = 1.21

Now add up the results:
.01+102.01+98.01+.81+1.21+3.61+9.61+8.41+.01+1.21 = 224.9

Now divide by the sample size minus 1

224.9 / 9 = 24.9889 <--the sample variance

Now calculate the **test statistic** using this equation:

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

"n" is the sample size
s^2 is the sample variance
σ^2 is the claimed population variance

[(10 - 1)24.9889] / 16 = **14.05626**

Does the test statistic fall in between the two critical values?
If YES, then do not reject the null hypothesis that the population variance is 16
if NO, then reject the null hypotheses that the population variance is 16.

Recall: Our two critical values are **3.3251 and 16.9190**
Our test statistic is **14.05626**

Our test statistic falls between the two critical values, so we do NOT reject the null hypothesis. Our sample of gears is consistent with a machine that is operating normally.