Unit 9, Video 2:
Comparing variances from two populations

Examples of comparing two variances:

Example 1: You think that adults in Detroit have more variable weights than adults in Houston.

Example 2: You think that your LCD plant in Mexico produces LCD panels with more precise thickness than your LCD plant in China

Important mathematical note:

Variance is the cousin of *Standard Deviation*

Variance = (standard deviation)^2

$\sqrt{\text{variance}}$ = standard deviation

Important mathematical note #2:

In a ratio, the numerator is the top number and the denominator is the bottom number:

$$\frac{\text{numerator}}{\text{denominator}}$$

Hypothesis tests comparing variances:
One-tailed or two-tailed?

This is a two-tailed test:

Null Hypothesis: Men's IQ scores have the same standard deviation as women's IQ scores

Alternate hypothesis: Men's IQ scores do not have the same standard deviation as women's IQ scores

$\neq$

Hypothesis tests comparing variances:
One-tailed or two-tailed?

This is a one-tailed test:

Null Hypothesis: Monthly rainfall in Honolulu has the same variation as monthly rainfall in Houston

Alternate hypothesis: Monthly rainfall in Houston has a larger variation than monthly rainfall in Honolulu

$>$ or $<$

Steps 1-4 to do a hypothesis test comparing two population variances:

1. State the null hypothesis and the alternate hypothesis

2. Determine whether your test is 1-tailed or 2-tailed.

3. Determine the significance level of the test (unless it has been dictated to you)

4. Use the correct F-distribution table (Appendix H) to read the critical F-value from the table.

(Continued on next page)

Steps 5-7 to do a hypothesis test comparing two population variances:

5. (This step is only necessary if you have standard deviations but not variances for your samples. If you already have variances for your samples, then skip this step.) Square the standard deviations from your two samples so that you have variances for both samples.

6. Compute your test F-statistic. (This is a ratio--one sample's variance divided by the other sample's variance.)

7. If your test F-statistic is larger than the critical F-statistic then reject the null hypothesis. If not, then do not reject the null hypothesis.

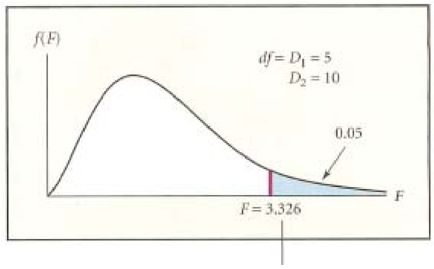
Which F-distribution table do I use?

There are 3 F-distribution tables in Appendix H.

Each table is 2-pages long.

APPENDIX H

F-Distribution Table: Upper 5% Probability (or 5% Area) Under *F*-Distribution Curve



Use the 5% table in these cases:

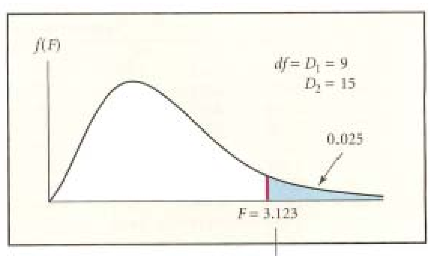
You are doing a 1-tailed test with .05 significance

or

You are doing a 2-tailed test with .10 significance

(continued)

F-Distribution Table: Upper 2.5% Probability (or 2.5% Area) Under *F*-Distribution Curve



Use the 2.5% table in these cases:

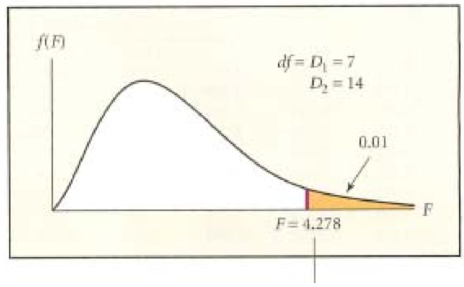
You are doing a 1-tailed test with .025 significance

or

You are doing a 2-tailed test with .05 significance

(continued)

F-Distribution Table:
Upper 1% Probability
(or 1% Area) Under
F-Distribution Curve



Use the 1% table in these cases:

You are doing a 1-tailed test with .01 significance

or

You are doing a 2-tailed test with .02 significance

How to calculate the test F-statistic

The test F-statistic is a ratio. One of your sample variances is the numerator; the other sample variance is the denominator.

Test F-statistic = $\dfrac{\text{One sample variance}}{\text{The other sample variance}}$

But which variance is the numerator and which is the denominator? (See next page.)

But which variance is the numerator and which is the denominator?

If you are doing a 2-tailed test, the numerator is always the larger sample variance.

If you are doing a 1-tailed test, the numerator is sample variance that the alternate hypothesis claims should be larger. (See next page for an example.)

If you are doing a 1-tailed test, the numerator is sample variance that the alternate hypothesis claims should be larger.

Example:
Null Hypothesis: Monthly rainfall in Honolulu has the same variation as monthly rainfall in Houston

Alternate hypothesis: Monthly rainfall in Houston has a larger variation than monthly rainfall in Honolulu

So: Put the Houston variance in the numerator (and the Honolulu variance in the denominator).

Now let's do a problem:

Dr. No thinks that male IQs have the same variability as female IQs. You think that female IQs have a lower variability than male IQs. You randomly sample 25 males and 21 females. The male IQ standard deviation in your sample is 20 points and the female IQ standard deviation in your sample is 15 points. Use your samples to test Dr. No's claim against your own claim using a 5% significance level.

1. State the null hypothesis and the alternate hypothesis.

Null hypothesis: male IQ variance = female IQ variance

Alternate hypothesis:
Female IQ variance < Male IQ variance

2. Determine whether your test is 1-tailed or 2-tailed.

Because there's a "<" sign in the alternate hypothesis it's a 1-tailed test.

3. Determine the significance level of the test (unless it has been dictated to you)

We have been given a 5% significance level

4. Use the correct F-distribution table (Appendix H) to read the critical F-value from the table.

Because our alternate hypothesis claims that the male IQ variance is larger, the male IQ variance goes in the numerator

$$\frac{\text{Male IQ Variance}}{\text{Female IQ variance}}$$

numerator df = male sample size − 1 = 25−1 = 24

denominator df = female sample size − 1 = 21−1 = 20

What the heck is the df in the table?

The "Numerator df" determines the column to read. This df equals the numerator sample size minus one.

The "Denominator df" determines the row to read. This df equals the denominator sample size minus one.

df = n -1

DENOMINATOR $df = D_2$	NUMERATOR $df = D_1$						
	24	30	40	50	100	200	300
1	249.052	250.096	251.144	251.774	253.043	253.676	253.887
2	19.454	19.463	19.471	19.476	19.486	19.491	19.492
3	8.638	8.617	8.594	8.581	8.554	8.540	8.536
4	5.774	5.746	5.717	5.699	5.664	5.646	5.640
5	4.527	4.496	4.464	4.444	4.405	4.385	4.378
6	3.841	3.808	3.774	3.754	3.712	3.690	3.683
7	3.410	3.376	3.340	3.319	3.275	3.252	3.245
8	3.115	3.079	3.043	3.020	2.975	2.951	2.943
9	2.900	2.864	2.826	2.803	2.756	2.731	2.723
10	2.737	2.700	2.661	2.637	2.588	2.563	2.555
11	2.609	2.570	2.531	2.507	2.457	2.431	2.422
12	2.505	2.466	2.426	2.401	2.350	2.323	2.314
13	2.420	2.380	2.339	2.314	2.261	2.234	2.225
14	2.349	2.308	2.266	2.241	2.187	2.159	2.150
15	2.288	2.247	2.204	2.178	2.123	2.095	2.085
16	2.235	2.194	2.151	2.124	2.068	2.039	2.030
17	2.190	2.148	2.104	2.077	2.020	1.991	1.981
18	2.150	2.107	2.063	2.035	1.978	1.948	1.938
19	2.114	2.071	2.026	1.999	1.940	1.910	1.899
20	2.082	2.039	1.994	1.966	1.907	1.875	1.865
24	1.984	1.939	1.892	1.863	1.800	1.768	1.756
30	1.887	1.841	1.792	1.761	1.695	1.660	1.647
40	1.793	1.744	1.693	1.660	1.589	1.551	1.537
50	1.737	1.687	1.634	1.599	1.525	1.484	1.469
100	1.627	1.573	1.515	1.477	1.392	1.342	1.323
200	1.572	1.516	1.455	1.415	1.321	1.263	1.240
300	1.554	1.497	1.435	1.393	1.296	1.234	1.210

2.082 is the critical F value

5. (This step is only necessary if you have standard deviations but not variances for your samples. If you already have variances for your samples, then skip this step.) Square the standard deviations from your two samples so that you have variances for both samples.

Male IQ variance = $20^2 = 400$

Female IQ variance = $15^2 = 225$

6. Compute your test F-statistic. (This is a ratio--one sample's variance divided by the other sample's variance.)

$$\frac{\text{Male IQ variance}}{\text{Female IQ variance}} = \frac{400}{225} = 1.77778$$

7. If your test F-statistic is larger than the critical F-statistic then reject the null hypothesis. If not, then do not reject the null hypothesis.

Critical F: 2.082

Test F-statistic: 1.77777777777777777777778

Because our test F-statistic is not larger than the critical F, we do not reject the null hypothesis.

So we cannot reject Dr. No's claim that male IQ variability is the same as female IQ variability.