Business Statistics: A Decision-Making Approach

7th Edition

Chapter 19

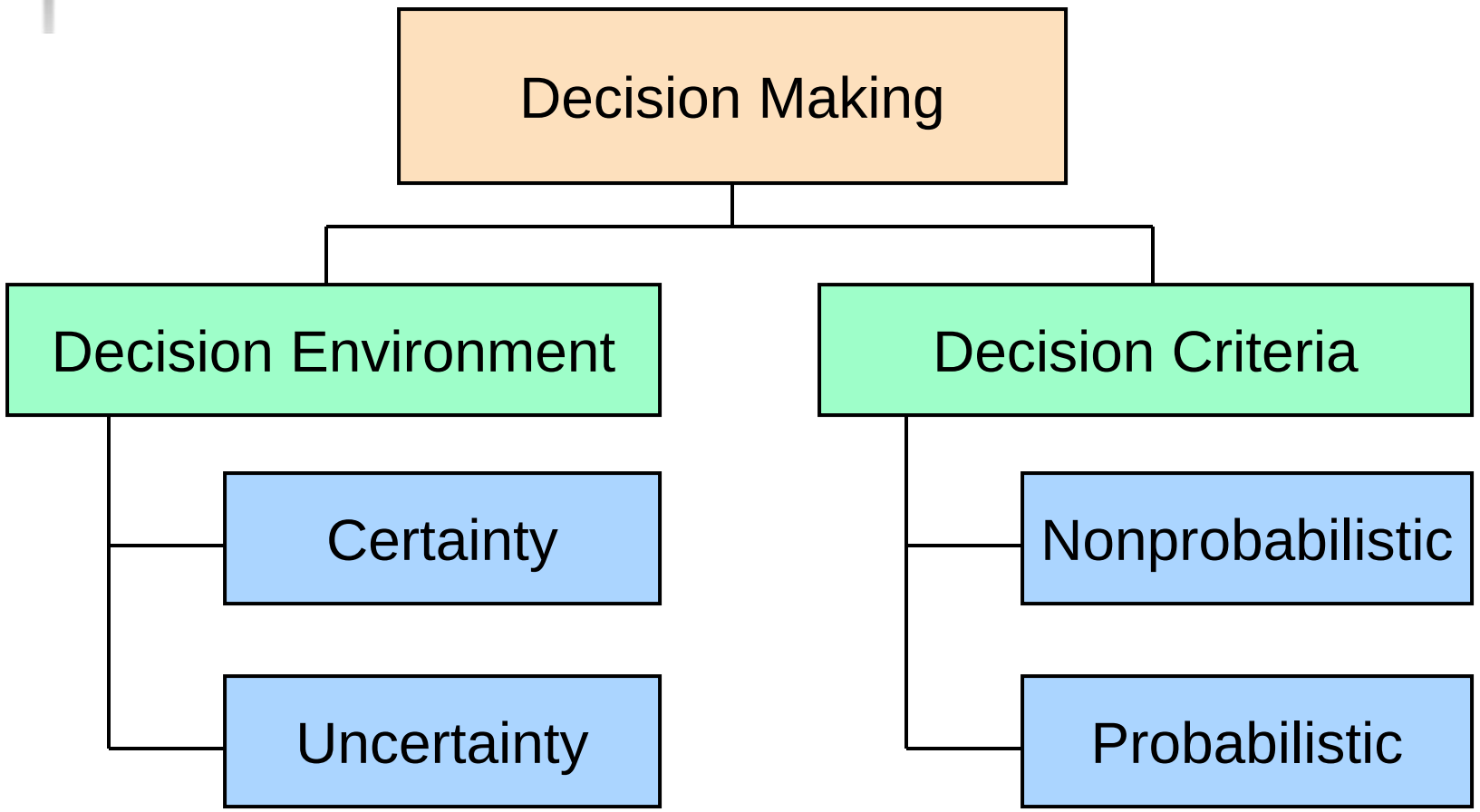
Introduction to Decision Analysis

Chapter Goals

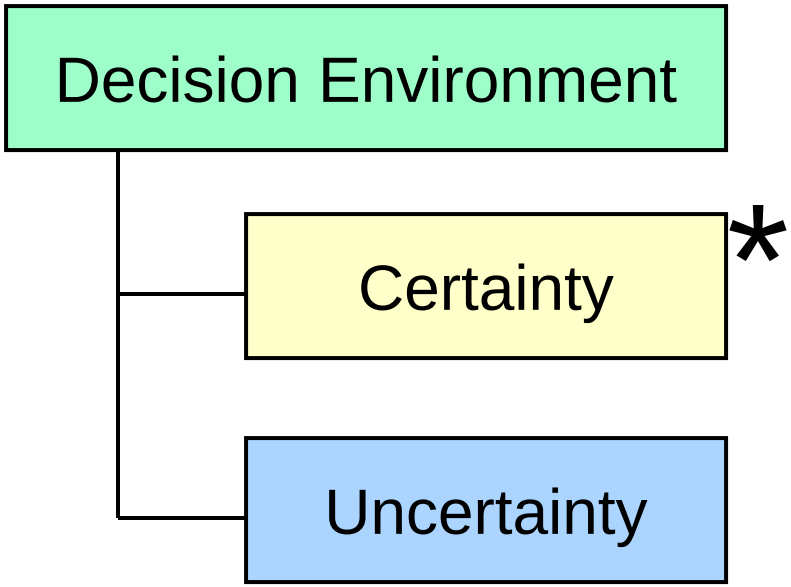
After completing this chapter, you should be able to:

- Describe the decision environments of certainty and uncertainty
- Construct a payoff table and an opportunity-loss table
- Define and apply the expected value criterion for decision making
- Compute the value of perfect information
- Develop and use decision trees for decision making

Decision Making Overview



The Decision Environment



Certainty: The results of decision alternatives are known

Example:

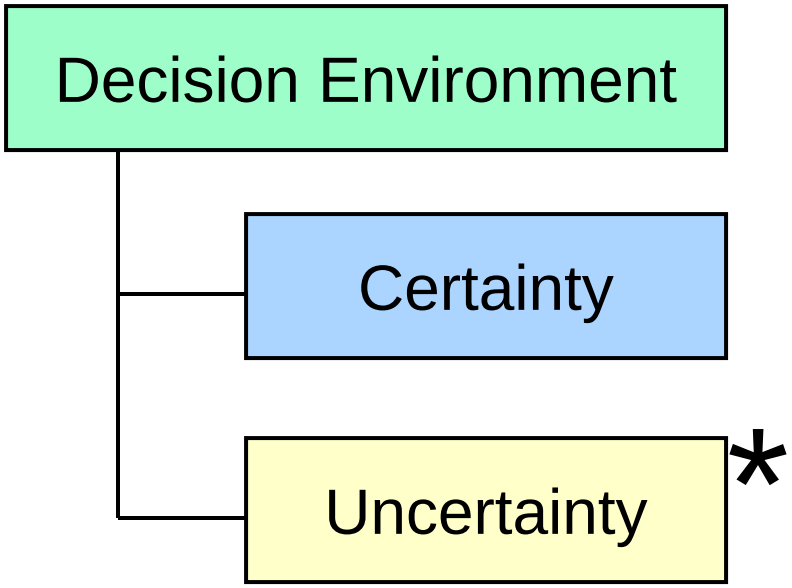
Must print 10,000 color brochures

Offset press A: $2,000 fixed cost
+ $.24 per page

Offset press B: $3,000 fixed cost
+ $.12 per page

The Decision Environment

(continued)



Uncertainty: The outcome that will occur after a choice is unknown

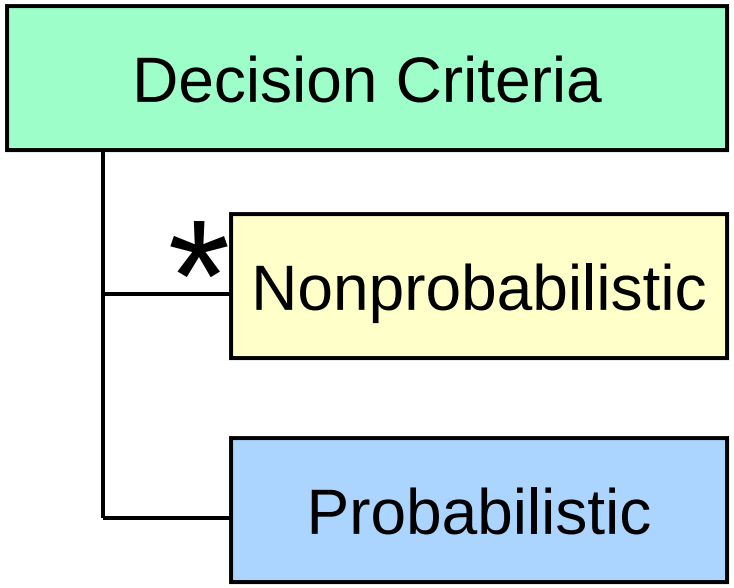
Example:

You must decide to buy an item now or wait. If you buy now the price is $2,000. If you wait the price may drop to $1,500 or rise to $2,200. There also may be a new model available later with better features.

Decision Criteria

Nonprobabilistic Decision Criteria: Decision rules that can be applied if the probabilities of uncertain events are not known.

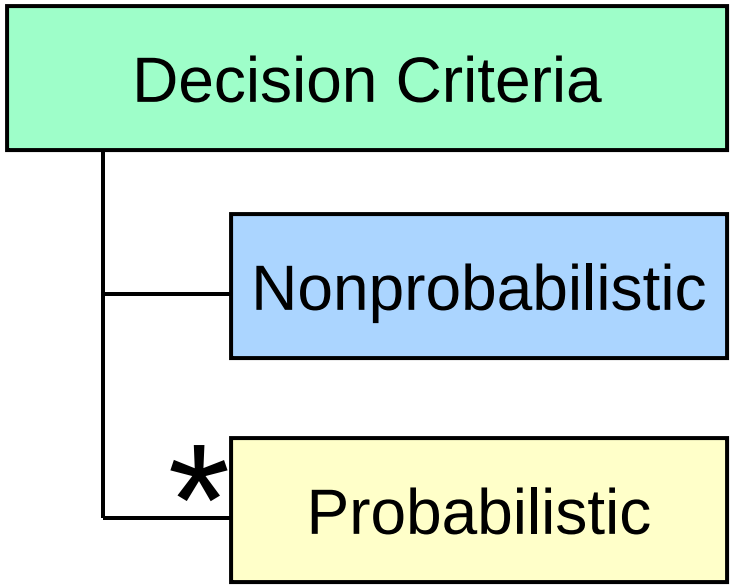
- maximax criterion
- maximin criterion
- minimax regret criterion



Decision Criteria

(continued)

Probabilistic Decision Criteria:
Consider the probabilities of uncertain events and select an alternative to maximize the expected payoff of minimize the expected loss



- maximize expected value
- minimize expected opportunity loss

A Payoff Table

A payoff table shows alternatives, states of nature, and payoffs

Investment Choice (Alternatives)	**Profit in $1,000's** (States of Nature)		
	Strong Economy	Stable Economy	Weak Economy
Large factory	200	50	-120
Average factory	90	120	-30
Small factory	40	30	20

Maximax Solution

The maximax criterion (an optimistic approach):

1. For each option, find the maximum payoff

Investment Choice (Alternatives)	**Profit in $1,000's** (States of Nature)			1. Maximum Profit
	Strong Economy	Stable Economy	Weak Economy	
Large factory	200	50	-120	→ 200
Average factory	90	120	-30	→ 120
Small factory	40	30	20	→ 40

Maximax Solution

(continued)

The maximax criterion (an optimistic approach):

1. For each option, find the maximum payoff
2. Choose the option with the greatest maximum payoff

Investment Choice (Alternatives)	**Profit in $1,000's** (States of Nature)			1. Maximum Profit
	Strong Economy	Stable Economy	Weak Economy	
Large factory	200	50	-120	200
Average factory	90	120	-30	120
Small factory	40	30	20	40

2. Greatest maximum is to choose **Large factory**

Maximin Solution

The maximin criterion (a pessimistic approach):

1. For each option, find the minimum payoff

Investment Choice (Alternatives)	**Profit in $1,000's** (States of Nature)			1. Minimum Profit
	Strong Economy	Stable Economy	Weak Economy	
Large factory	200	50	-120	→ -120
Average factory	90	120	-30	→ -30
Small factory	40	30	20	→ 20

Maximin Solution

(continued)

The maximin criterion (a pessimistic approach):

1. For each option, find the minimum payoff
2. Choose the option with the greatest minimum payoff

Investment Choice (Alternatives)	**Profit in $1,000's** (States of Nature)			1. Minimum Profit	2.
	Strong Economy	Stable Economy	Weak Economy		
Large factory	200	50	-120	-120	
Average factory	90	120	-30	-30	
Small factory	40	30	20	20	Greatest minimum is to choose **Small factory**

Opportunity Loss

Opportunity loss is the difference between an actual payoff for a decision and the optimal payoff for that state of nature

Investment Choice (Alternatives)	**Profit in $1,000's** (States of Nature)		
	Strong Economy	Stable Economy	Weak Economy
Large factory	200	50	-120
Average factory	90	120	-30
Small factory	40	30	20

Payoff Table

The choice "Average factory" has payoff 90 for "Strong Economy". Given "Strong Economy", **the choice of "Large factory" would have given a payoff of 200, or 110 higher**. Opportunity loss = 110 for this cell.

Opportunity Loss

(continued)

Investment Choice (Alternatives)	**Profit in $1,000's** (States of Nature)		
	Strong Economy	Stable Economy	Weak Economy
Large factory	200	50	-120
Average factory	90	120	-30
Small factory	40	30	20

Payoff Table

Opportunity Loss Table

Investment Choice (Alternatives)	**Opportunity Loss in $1,000's** (States of Nature)		
	Strong Economy	Stable Economy	Weak Economy
Large factory	0	70	140
Average factory	110	0	50
Small factory	160	90	0

Minimax Regret Solution

The minimax regret criterion:

1. For each alternative, find the maximum opportunity loss (or “regret”)

Opportunity Loss Table

Investment Choice (Alternatives)	**Opportunity Loss in $1,000’s** (States of Nature)			1. Maximum Op. Loss
	Strong Economy	Stable Economy	Weak Economy	
Large factory	0	70	140	→ 140
Average factory	110	0	50	→ 110
Small factory	160	90	0	→ 160

Minimax Regret Solution

(continued)

The minimax regret criterion:

1. For each alternative, find the maximum opportunity loss (or “regret”)
2. Choose the option with the smallest maximum loss

Opportunity Loss Table

Investment Choice (Alternatives)	**Opportunity Loss in $1,000’s** (States of Nature)			1. Maximum Op. Loss	2.
	Strong Economy	Stable Economy	Weak Economy		
Large factory	0	70	140	140	Smallest maximum loss is to choose **Average factory**
Average factory	110	0	50	110	
Small factory	160	90	0	160	

Expected Value Solution

- The expected value is the weighted average payoff, given specified probabilities for each state of nature

Investment Choice (Alternatives)	**Profit in $1,000's** (States of Nature)		
	Strong Economy **(.3)**	Stable Economy **(.5)**	Weak Economy **(.2)**
Large factory	200	50	-120
Average factory	90	120	-30
Small factory	40	30	20

Suppose these probabilities have been assessed for these states of nature

Expected Value Solution

(continued)

Investment Choice (Alternatives)	Profit in $1,000's (States of Nature)		
	Strong Economy (.3)	Stable Economy (.5)	Weak Economy (.2)
Large factory	200	50	-120
Average factory	90	120	-30
Small factory	40	30	20

Expected Values

61

81

31

Maximize expected value by choosing **Average factory**

Example: EV (Average factory) = 90(.3) + 120(.5) + (-30)(.2)
= 81

Expected Opportunity Loss Solution

Opportunity Loss Table

Investment Choice (Alternatives)	Opportunity Loss in $1,000's (States of Nature)			Expected Op. Loss (EOL)
	Strong Economy (.3)	Stable Economy (.5)	Weak Economy (.2)	
Large factory	0	70	140	63
Average factory	110	0	50	43
Small factory	160	90	0	93

Minimize expected op. loss by choosing **Average factory**

Example: EOL (Large factory) = 0(.3) + 70(.5) + (140)(.2)
= 63

Cost of Uncertainty

- Cost of Uncertainty (also called Expected Value of Perfect Information, or EVPI)
- Cost of Uncertainty

  = Expected Value Under Certainty (EVUC)

  – Expected Value without information (EV)

so:

Expected Cost of Uncertainty
= EVPI
= EVUC – EV

Expected Value Under Certainty

- Expected Value Under Certainty (EVUC):

EVUC = expected value of the best decision, given perfect information

Investment Choice (Alternatives)	**Profit in $1,000's** (States of Nature)		
	Strong Economy **(.3)**	Stable Economy **(.5)**	Weak Economy **(.2)**
Large factory	200	50	-120
Average factory	90	120	-30
Small factory	40	30	20
	200	120	20

Example: Best decision given "Strong Economy" is "Large factory"

Expected Value Under Certainty

(continued)

Investment Choice (Alternatives)	Profit in $1,000's (States of Nature)		
	Strong Economy (.3)	Stable Economy (.5)	Weak Economy (.2)
Large factory	200	50	-120
Average factory	90	120	-30
Small factory	40	30	20

200	120	20

- Now weight these outcomes with their probabilities to find EVUC:

$$EVUC = 200(.3)+120(.5)+20(.2)$$
$$= 124$$

Cost of Uncertainty Solution

- **Cost of Uncertainty (EVPI)**
  = Expected Value Under Certainty (EVUC)
  – Expected Value without information (EV)

Recall: EVUC = 124

EV is maximized by choosing “Average factory”, where EV = 81

so: **EVPI = EVUC – EV**
= 124 – 81
= 43

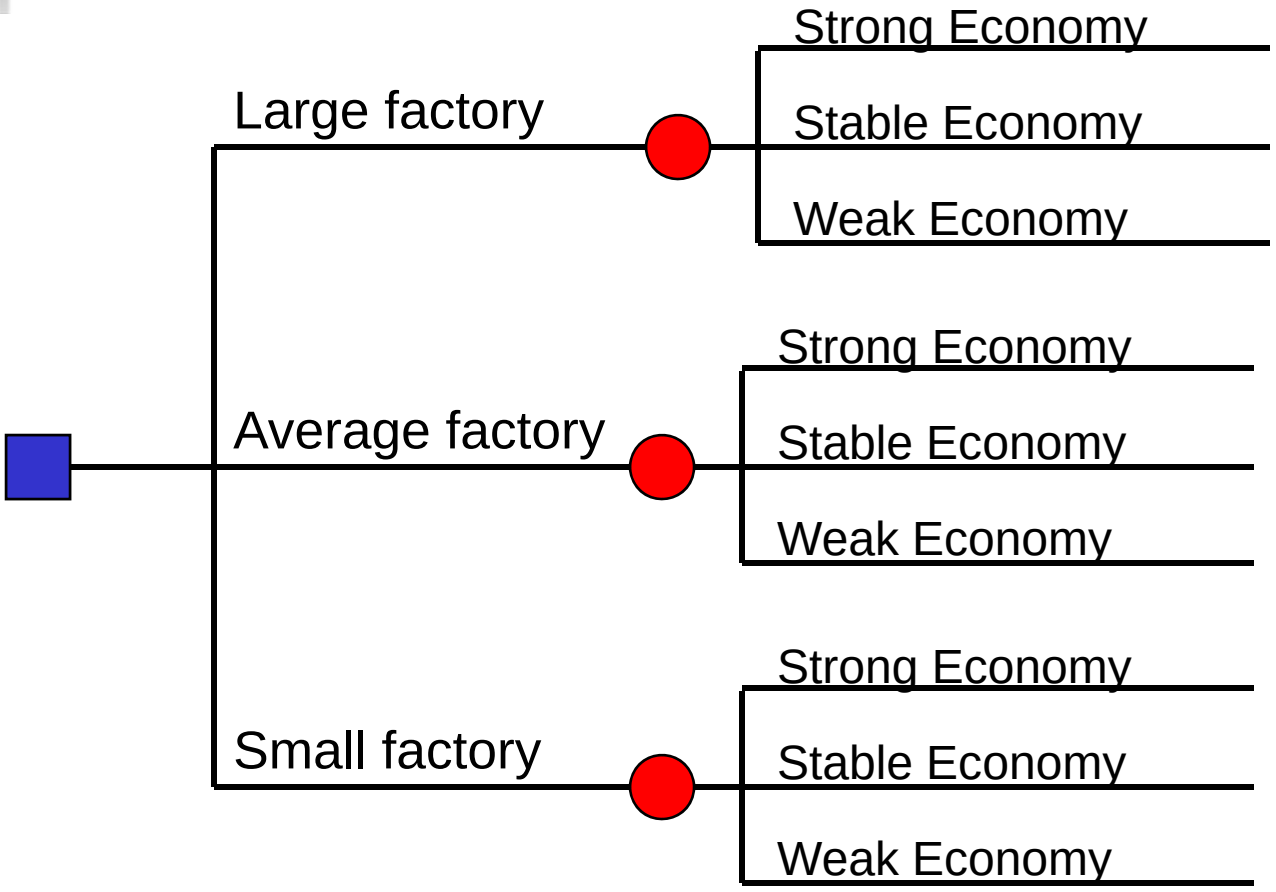
Decision Tree Analysis

- A Decision tree shows a decision problem, beginning with the initial decision and ending will all possible outcomes and payoffs.

Use a square to denote decision nodes

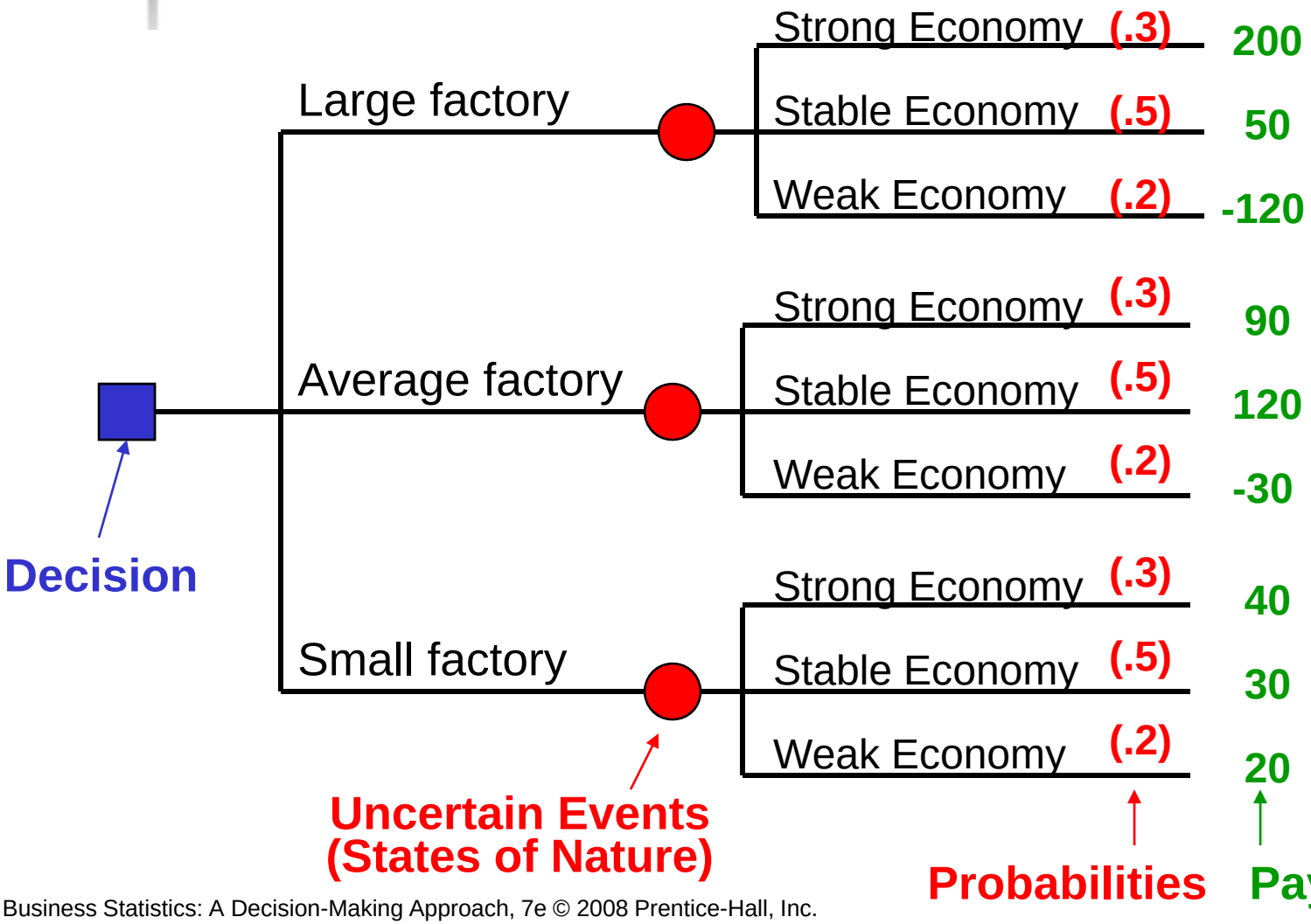
Use a circle to denote uncertain events

Sample Decision Tree

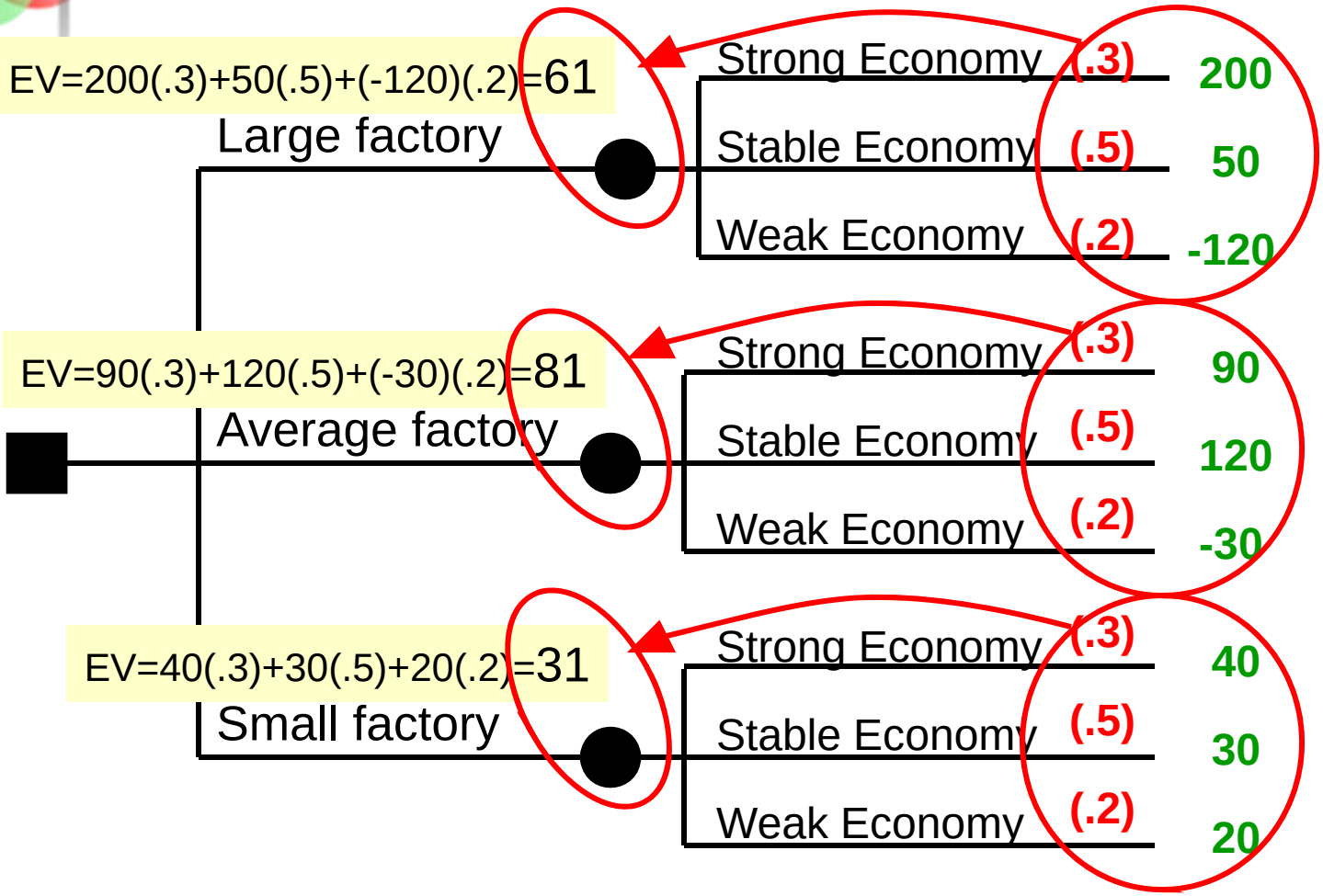


Add Probabilities and Payoffs

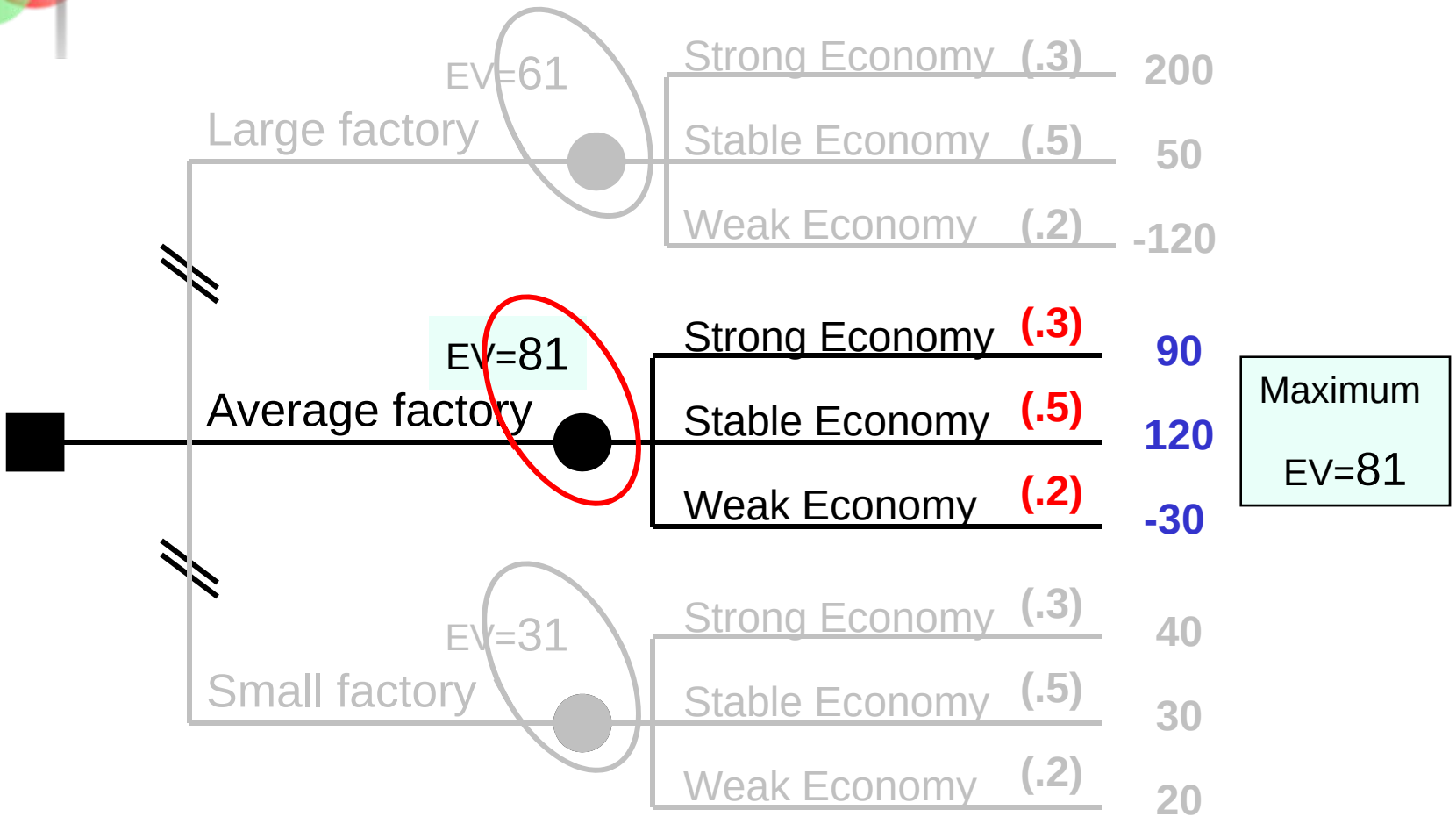
(continued)



Fold Back the Tree



Make the Decision



Chapter Summary

- Examined decision making environments
  - certainty and uncertainty
- Reviewed decision making criteria
  - nonprobabilistic: maximax, maximin, minimax regret
  - probabilistic: expected value, expected opp. loss
- Computed the Cost of Uncertainty (EVPI)
- Developed decision trees and applied them to decision problems