Analysis of Monopoly (With No Price Discrimination)

Contents:

Introduction:

Firms operate within their market, which consists of:

Supply side: all of the firms producing similar products
Demand side: all buyers willing to purchase the products

Markets differ; the auto market is far different from the tomato market, for example. Thus economists separate markets into 4 categories:

--perfect competition
--monopolistic competition
--oligopoly
--monopoly.

Behold the qualities of each category of market below:

Perfect competition: There are many, many small sellers in the market (technically, there must be an infinite number of sellers), each of whom produces an **identical** product. It is very easy for new sellers to enter this market, and it is easy for existing sellers to leave the market.

Examples: There are no real world examples of perfectly competitive markets. Some agricultural markets come close. The stock market comes close.

Monopolistic competition: There are many small sellers in the market, each of whom produces a slightly **different** product. It is very easy for new sellers to enter this market, and it is easy for existing sellers to leave the market.

Examples: Chinese restaurants, lawyer services, plumbing services, haircuts.

Oligopoly: There are a few large sellers that dominate the market, each of whom produces either an identical product or a slightly different product. It is difficult for new sellers to enter this market, and it can be difficult for existing sellers to leave the market.

Examples: Oil refining, autos, copper, airlines, computers.

Monopoly: There is one large seller with no direct competition. It is extremely difficult for new sellers to enter this market, and it can be difficult for the existing seller to leave the market.

Examples: U.S. letter delivery, markets for patented products, residential natural gas service.

In this set of notes, we shall examine the attributes of a **monopoly**. We begin by examining the firm's revenues.

IMPORTANT NOTE: NO PRICE DISCRIMINATION. We shall assume in this set of notes that the monopoly charges the same price for each unit of output; hence there is *no price discrimination*. (In another set of notes notes, we'll see that the firm with market power can usually increase profits using price discrimination (if it's legal and feasible).)

MARKET POWER: A firm is said to have *market power* if it can increase its price and not lose all of its customers; graphically, this means that the firm's demand curve is not horizontal. Some firms have more market power than others; the ultimate in market power is if a hypothetical firm, when it raises its price, loses NONE of its customers; graphically this is a vertical demand curve.

A perfectly competitive firm has no market power; firms in other markets have at least some degree of market power. A monopoly has a lot of market power, but it is rare indeed for even a monopoly to have a vertical demand curve.

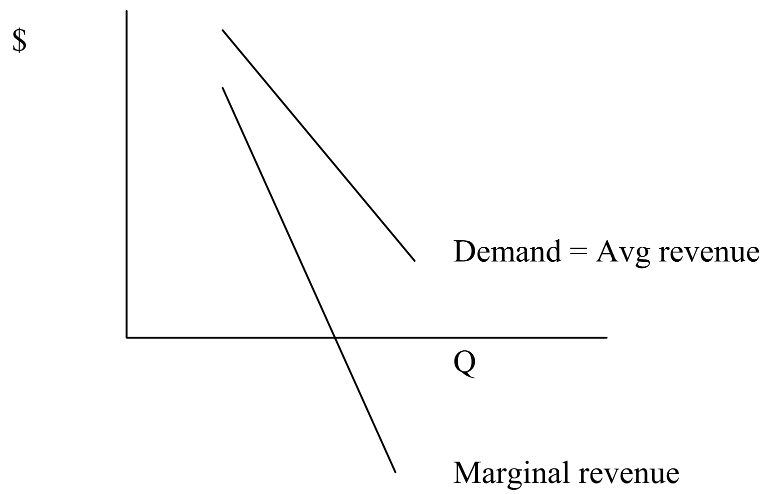
Revenues of a monopoly

In this type of market, the market price is established by the interaction of buyers in total and the *single seller*. Since there is a single seller in the market, the monopoly can raise its price and lose only a few buyers. (Suppose, for example, that you own the only gas station within 200 miles. You can raise your gas prices and still sell a lot of gas.)

Hence a monopoly is a price-*maker*; it can set its own selling price. At a higher price, it has a bit lower demand, and at lower prices it has a bit higher demand. Implication: a monopoly has a steep, downward-sloping demand curve. (It is theoretically possible, though unlikely, for a monopoly to have a completely vertical demand curve.)

Let's illustrate a monopoly's revenues on the graph below:

Revenues of a monopoly



Note 1 about the graphs: Since the seller loses a few buyers as it raises its price, it has a downward-sloping and steep demand curve.

Note 2 about the graphs: Recall that the price equals the firm's average revenue; hence the firm's average revenue is falling as Q rises. This means that the firm's marginal revenue is falling and is lower than the price (just as a student who keeps getting a lower and lower test score drives his/her average lower and lower). Hence the marginal revenue curve is below the demand curve. (See the notes file "mana-profit" for a more thorough explanation.)

Note 3 about the graphs: With a monopoly, the graph of the demand for a firm's product is equivalent to the market demand, since the firm is the only seller in the market. It is sometimes said that with a monopoly "the firm is the industry."

If we know a monopoly's demand curve, we know all we need about its total, average, and marginal revenue. Observe:

Example 1: The first two columns in the table below indicate how the demand for a monopoly's product varies with the price that it charges. Show how an the monopoly's total, average, and marginal revenue vary with its level of production for Q = 0 to 6

(Don't forget: we assume that each unit must sell for the same price, e.g. if the firm charges $3 per unit then each of the 5 units sells for $3 each.)

Q	Price (= average revenue)	Total revenue (= P x Q)	Marginal revenue
0	8	0	na
1	7	7	7
2	6	12	5
3	5	15	3
4	4	16	1
5	3	15	-1
6	2	12	-3

Example 2: The market demand for goo (a monopoly market) is P = 100 – 2Q. Derive equations representing the goo seller's total, average, and marginal revenue.

AR = price → AR = 100 – 2Q

TR = p x q → TR = (100 – 2Q)Q → TR = 100Q – $2Q^2$

MR = $\frac{dTR}{dQ}$ → MR = 100 – 4Q

("AR" is average revenue, "TR" is total revenue, and "MR" is marginal revenue.)

(Useful **shortcut**: notice that the MR curve has the same vertical intercept as the demand curve, and is twice as steep. This will always be true for a linear demand curve.)

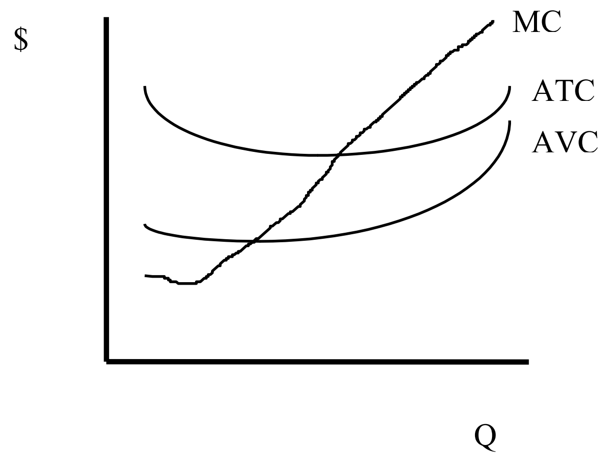
(Note: If you are given a demand equation with Q on the left hand side, then you must first rearrange it to get P by itself on the left hand side, in order to get the AR, TR, and MR equations. Example: If given Q = 5 – 2P, you must rearrange to get P = 2.5 - .5Q)

<u>Short Run Costs of A Monopoly</u>
In the notes file "mana-costs" we discussed short run costs. I review the analysis here, to refresh your recollection:

GRAPH: A fairly realistic representation of any firm's short run costs:

Any and all firms face rising MC, AVC, and ATC, beyond some level of Q, due to the law of diminishing returns. Hence it is reasonable to represent any and all firms' short run costs with the graph below:

Any firm's short run costs



Note on the graph above how ATC, AVC, and MC all eventually rise. Note also that *MC intersects AVC and ATC at the minimum points of both the AVC and ATC curves* (the bottom of each "U".)

EQUATION: Example of a short run cost function:

$TC = 1000 + 5Q^2$

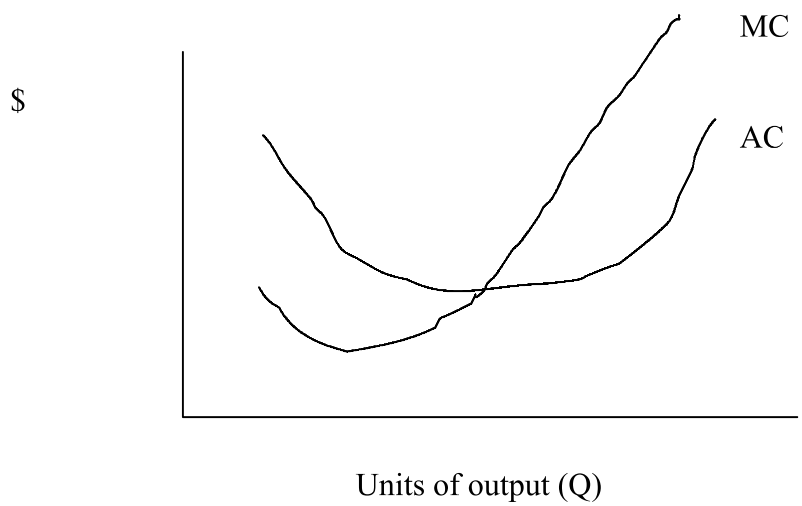
The "1000" in the above equation is fixed costs; the "$5Q^2$" is variable costs.

Long Run Costs of A Monopoly

In the notes file "mana-costs" we discussed long run costs. I repeat some of the discussion here.

Graph: Different shapes for a firm's long run cost curves are possible, depending upon the importance of scale economies in the production of the firm's product. I will sometimes represent a monopoly's long run costs with this graph: (We abbreviate long run marginal cost with "MC," and long run average cost with "AC.")

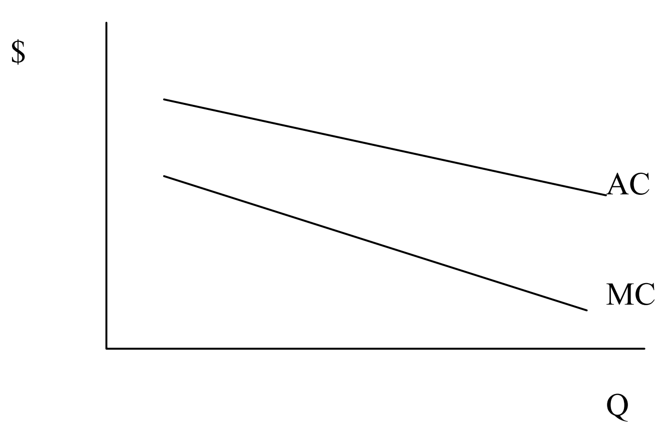
One Possible Depiction of a monopoly's long run costs



NATURAL MONOPOLY: Suppose that economies of scale are so large in a market that it is cheaper for 1 firm to provide all of the market's production, relative to a bunch of smaller firms. This market is called a *natural monopoly*. (It is difficult to think of a real world market where this is true. It used to be argued that electricity provision was a natural monopoly, under the reasoning that multiple firms would have to string multiple electric lines throughout the city. In New York City early in this century, there were as many as five sets of power lines running throughout the city—1 set for each of the five power companies. Clearly, if it were necessary for each power company to string its own set of power lines, then it would be less costly for 1 firm to produce all of New York City's power, and indeed this was the case in the early 20th century. Modern technology, however, allows multiple power providers to use the same set of power lines, so electricity provision is no longer a natural monopoly.)

One would graph the long run cost curves of a natural monopoly like this:

A natural monopoly's long run costs



Equation: here is an example of a long run cost equation:

$TC = 6Q^2$

(Note that there is no constant term, since there are no fixed costs in the long run.)

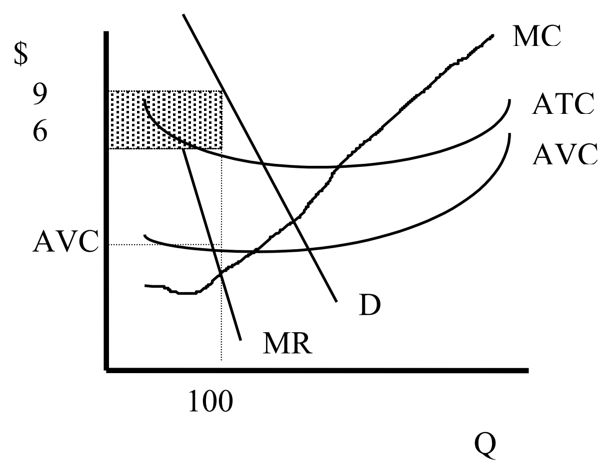
Profit-maximizing short run strategy for a monopoly:

Recall that according to the profit-maximizing rule (as fully discussed in the notes "mana-profit," a firm should:

→ produce the level of output where MR = MC, unless P<AVC at that level of output (in which case the firm should shut down, producing Q=0).

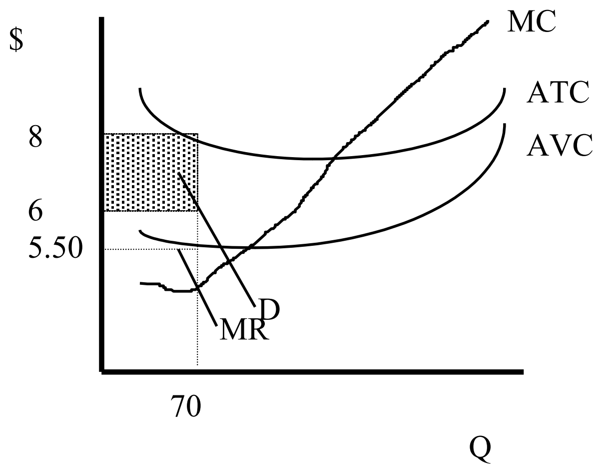
Let's look at three graphs, each representing a monopoly in the short run. The major thing differentiating the graphs is the height of the demand curve and MR curve.

Graph 1: A monopoly making a profit in the short run



The above firm is making a profit; the price of \$9 is above the average total cost of \$6. The firm is making \$9-\$6 = \$3 per unit sold, and they're selling 100 units, making total profit = \$300 (the area of the shaded rectangle).

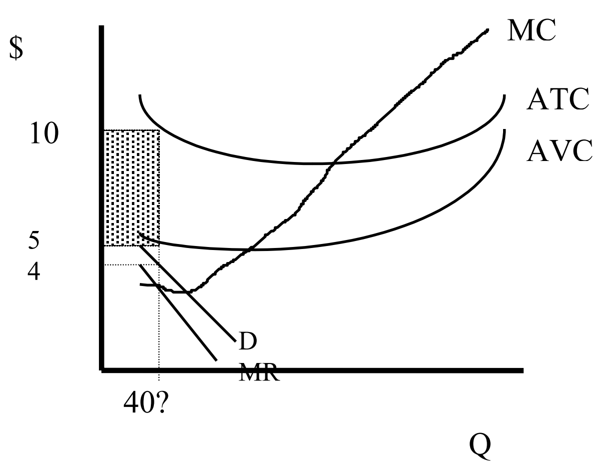
Graph 2: A monopoly operating at a loss in the short run



The above firm is operating at a loss; the price of $6 is below the average total cost of $8. The firm is losing $8-$6 = $2 per unit sold, and they're selling 70 units, making total loss = $140 (the area of the shaded rectangle).

Note that the firm would do worse if it shut down, since P>AVC. Indeed, the firm would lose (8-5.50) x 70 = $175, its total fixed costs, if it shut down.

Graph 3: A monopoly in the short run which is shut down.



The above firm does best if its shuts down, since P<AVC. If it shuts down it loses its fixed costs, (10-5) x 40 = $200. If it were dumb enough to operate, it would lose (10-4) x 40 = $240.

Two algebraic examples:

1st example: A monopoly has cost equation $TC = 100 + Q^2$.
Market demand is $P = 100 - Q$. Price discrimination is illegal.
What is the firm's profit-maximizing strategy?

Set MR = MC

Note : $MR= 100 – 2Q$ (twice as steep as demand curve, Same intercept. See **shortcut** on p. 4)

Note: $MC = \frac{dTC}{dQ} = 2Q$

$100 – 2Q = 2Q$

$Q = 25$

Plug Q=25 into demand equation to get P:

$P = 100 – Q = 100 – 25 = 75$

Hence the firm should produce 25 units and charge 75 each. Its profit will be:

Profit = revenue – cost

= p x q – TC

$= 75 \times 25 – (100 + 25^2)$

= 1875 – 725 = $1150

2nd example: A monopoly has cost equation $TC = 100 + Q^2$.
Market demand is $P = 8 - Q$. Price discrimination is illegal.
What is the firm's profit-maximizing strategy?

Set MR = MC

Note : $MR= 8 – 2Q$ (twice as steep as demand curve, Same intercept. See **shortcut** on p. 4)

Note: $MC = \frac{dTC}{dQ} = 2Q$

$8 – 2Q = 2Q$

$Q = 2$

Plug Q=2 into demand equation to get P:

$P = 8 – Q = 8 – 2 = 6$

Hence the firm should produce 2 units and charge 6 each. Its profit will be:

Profit = revenue – cost

= p x q – TC

= 6 x 2 – (100 + 2^2)

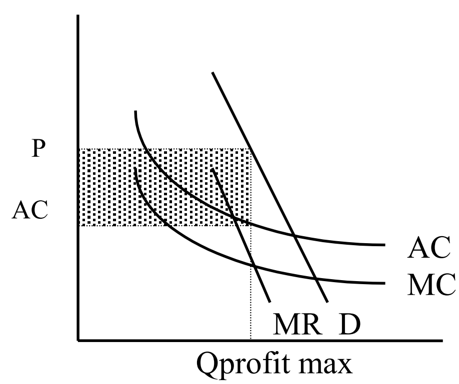
= 12 – 104 = $-92 a loss of 92

This is still better than shutting down, in which case the firm would lose its fixed costs of $100.

Long run monopoly: If unregulated, profits persist into long run!!!

Recall that intense competition eliminates profits in a perfectly competitive market. A monopoly has no direct competition, and if it can maintain the barriers that prevent competition into the long run, and if it can remain unregulated by government, then its profits will persist into the long run. Here's an example, using a natural monopoly (though this can also be true for non-natural monopolies):

Graph: An unregulated natural monopoly in the long run, with no price discrimination:



We have seen profit-maximizing strategies under monopoly. But is profit-seeking "good" for society? Under monopoly, it is NOT. Read on.

Monopoly, Worldly Philosophers, and Economic Efficiency

Suppose you are a worldly philosopher, and want to measure if production in your economy is serving your citizens. You might have a measure as follows:

Social (or allocative) Efficiency: Extra production occurs as long as society feels that the benefit from the extra production exceeds the cost of extra production. Production does not occur if it costs more to produce the thing than the benefit of the production.

How do we measure the benefit of extra production? It is the PRICE that a buyer is willing to pay for the product!!! Why? Well, if someone is willing to pay for a product then they must be getting at least that much value from it.

How do we measure the cost of extra production? Marginal cost!!!

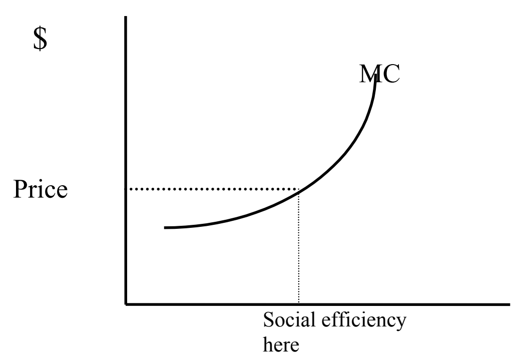
Summary of social efficiency:

If P > MC → more production is warranted, since the benefit to the buyer outweighs the cost to the producer

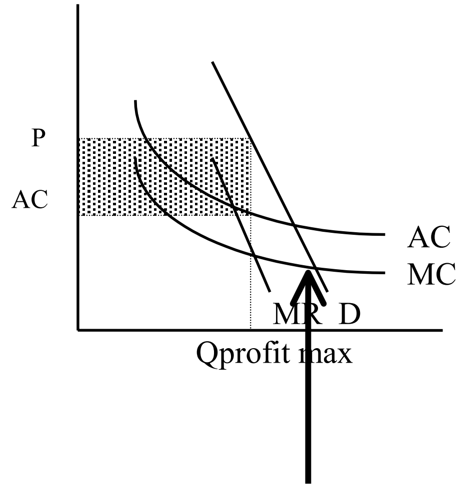
If P< MC → less production is warranted, since the benefit to the buyer is less than the cost to the producer

So if P=MC → socially efficient level of production!!!

See the graph below:



UNREGULATED MONOPOLY AIN'T SOCIALLY EFFICIENT: So, you're a worldly philosopher, and you want production to have qualities of social efficiency. Does an unregulated monopoly fit the bill? No! Check out the graph on page 10. The price is way above MC. Indeed, let's redraw that graph below, while inserting an indicator of the socially efficient quantity of production:



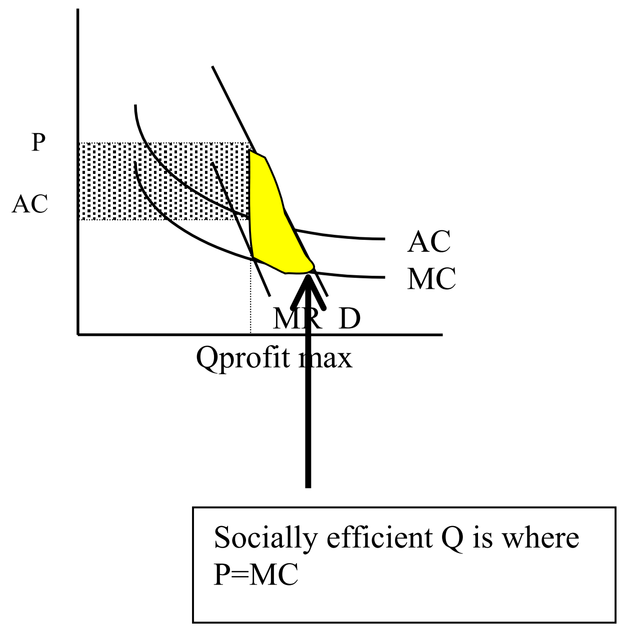
Socially efficient Q is where P=MC

Notice that the socially efficient quantity is higher than the profit-maximizing quantity.

Unregulated monopoly produces less than the socially efficient level of output

Measuring the inefficiency of unregulated monopoly (with no price discrimination): Deadweight Loss

How much does society lose because the unregulated monopoly produces too little output? Well, it loses the (marginal benefit – marginal cost) of the units that were not produced. Recall that marginal benefit is measured by the demand curve; and of course we have a marginal cost curve. Graphically, the lost benefit to society of unregulated monopoly = the area between the demand curve and the MC curve above the units that were not produced. It's the yellow shaded area below:



<u>Algebraic example of deadweight loss:</u>
A monopoly has demand curve P = 100 – Q and cost curve TC = Q^2. Price discrimination is illegal.
Calculate the deadweight loss of profit-maximizing production.

Solution strategy:
1st: Calculate profit-maximizing price, quantity, and marginal cost. Call them Q_{max}, P_{max}, and MC_{max}
2nd: Calculate the socially efficient level of Q where P=MC. Call it $Q_{efficient}$
3rd: Calculate area of triangle with base = ($Q_{efficient}$-Q_{max}) and height (P_{max}-MC_{max})

1st: Note: MR = 100 – 2Q (twice as steep as demand)

Note: $MC = \frac{dTC}{dQ} = 2Q$

MR = MC → 100 – 2Q = 2Q → Q_{max} = 25

Now plug in Q_{max} = 25 into the demand equation to get P_{max}:

P_{max} = 100 – 25 = 75

Now plug in Q_{max} = 25 into the marginal cost equation to get MC_{max}:

MC_{max} = 2(25) =50

(See how P_{max} is not equal to MC_{max}? This demonstrates that the profit-seeking monopoly does not produce a socially efficient level of output.)

2nd: Set demand = marginal cost to find the socially efficient quantity (where P=MC)

$100 - Q = 2Q \rightarrow 3Q = 100 \rightarrow Q_{efficient} = 33\ 1/3$

(Notice that the socially efficient quantity, 33 1/3, is way more than the profit-maximizing quantity, 25.)

3rd: Deadweight loss = area of triangle between Demand and MC above Q not produced

$= .5(Q_{efficient} - Q_{max})(P_{max}-MC_{max})$

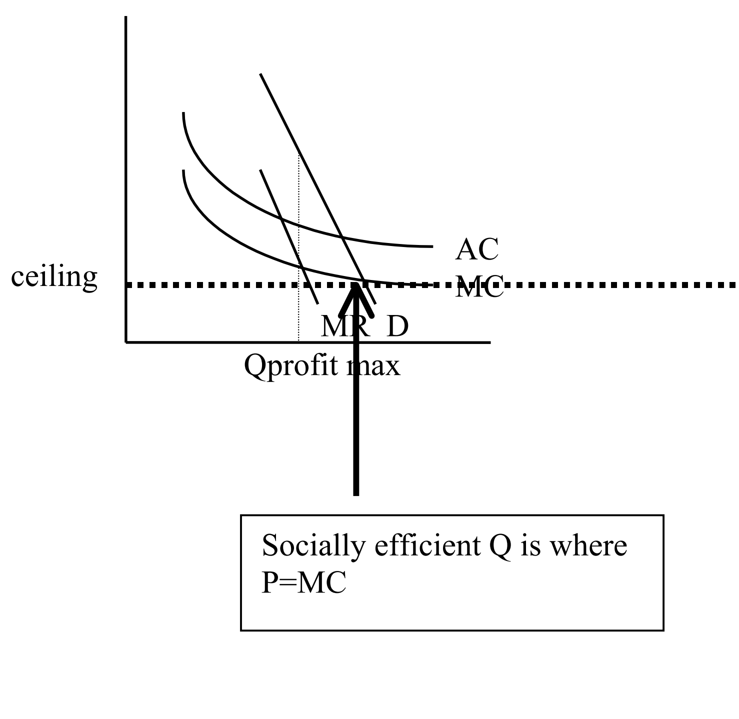
$= .5(33.33 - 25)(75 - 50) = \$\ 104.16$

(Interpretation of deadweight loss: If somehow one could get the monopoly to produce 33 1/3 units instead of 25 units, then society would achieve a net benefit worth $104.16.)

<u>Government regulation to reduce deadweight loss?</u>

In theory, government can set a price ceiling to reduce or eliminate the deadweight loss of unregulated monopoly (with no price discrimination). The price ceiling prevents the firm from pursuing its profit-maximizing strategy. Here's 2 possible price ceilings:

1. Set ceiling to ensure that P=MC.
Suppose government sets a price ceiling as in the graph below. This ensures that P = MC.

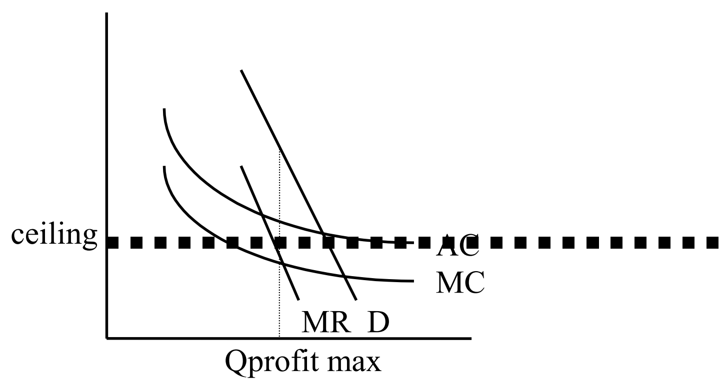


A price ceiling of the height of the heavy dashed line above ensures that production occurs at the socially efficient level. BUT WAIT! The price is below the firm's long run average cost at the socially efficient quantity! The firm would lose $$ and eventually go out of business, unless:

It is often necessary for government to provide a subsidy to a monopoly if it wishes the monopoly to produce the socially efficient level of output.

2nd: Set a break-even price ceiling

This is a more common practice. Monopoly prices are sometimes regulated to ensure a "fair" rate of return, which means zero economic profits. The ceiling would have to be set so that P=AC, as in the graph below:



At this ceiling no government subsidy is required. The deadweight loss is reduced but it is not eliminated.

A final word about government regulation of monopoly: In reality it is difficult for a government agency to know a monopoly's costs with great precision; hence finding the breakeven or socially efficient price ceiling is difficult. Many monopolies exist due to government protection. It is perhaps better for government to end this protection and allow competition to enter the industry; indeed, this has happened in many economies in the late 20th and early 21st centuries.

Profit-Maximizing Production With Multiple Plants:

(Note: though this section is in the "monopoly" notes, it is applicable to any type of firm that has multiple plants—oligopoly, monopolistic competition, and perfect competition too.)

Suppose a monopoly has two plants that produce the same item. How much should it produce at each plant? Answer: it should produce at each plant as long as the marginal cost of production at the plant is less than the marginal revenue generated by the sale of that production. (This assumes that the ultimate selling price exceeds AVC; if P<AVC, the firm should produce Q=0 at both plants.)

I shall do 2 examples—a tabular example and an algebraic example.

MULTI-PLANT PRODUCTION: TABULAR EXAMPLE (no price discrimination)

Here are 3 tables. One is the marginal cost of producing at plant 1. Another is the marginal cost of producing at plant 2. The other is the marginal revenue from sales of the good.

Q, plant 1	**MC, plant 1**
1	2
2	4
3	6
4	8
5	8.99
6	12
7	14
8	15
9	17

Q, plant 2	**MC, plant 2**
1	1
2	3
3	8.98
4	10
5	10.50
6	11
7	13
8	16
9	18

Q sold (= sum of outputs of plant 1 and plant 2)	MR
1	34
2	31
3	23
4	21
5	17
6	15
7	14
8	9
9	8
10	7
11	5
12	4
13	2
14	1
15	-1
16	-3
17	-5
18	-7
19	-9

(Note: the firm produces output at both plants, then sells the combined output to buyers. The buyers can't tell which plant the good is from, since the goods produced are identical at both plants.)

<u>Strategy for the above firm as it expands production</u>: As the firm expands Q, it should produce the extra unit of Q using the plant with the lowest marginal cost, and continue production as long as the marginal cost is lower than the marginal revenue. Voila:

1st unit of Q : Use plant 2. MC is 1. MR is 34. This adds $33 to profit.
2nd unit of Q : Use plant 1. MC is 2. MR is 31. This adds $29 to profit.
3rd unit of Q : Use plant 2. MC is 3. MR is 23. This adds $20 to profit.
4th unit of Q : Use plant 1. MC is 4. MR is 21. This adds $17 to profit.
5th unit of Q : Use plant 1. MC is 6. MR is 17. This adds $11 to profit.
6th unit of Q : Use plant 1. MC is 8. MR is 15. This adds $7 to profit.
7th unit of Q : Use plant 2. MC is 8.98. MR is 14. This adds $5.02 to profit.
8th unit of Q : Use plant 1. MC is 8.99. MR is 9. This adds $.01 to profit.

Summary: The firm produces and sells 8 units of product. It uses plant 1 to produce 5 of them and plant 2 to produce 3 of them.

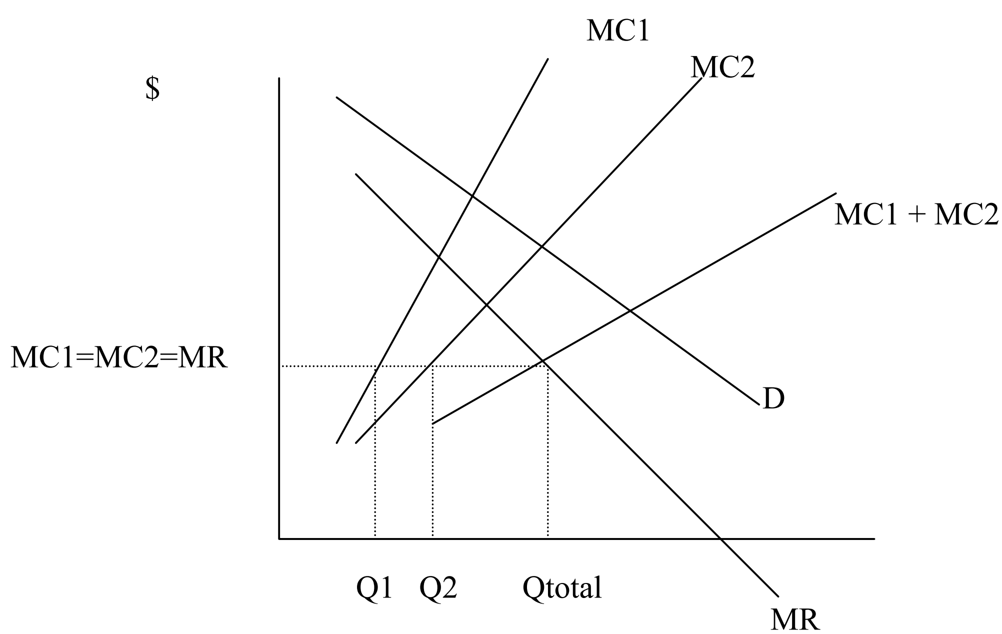
MULTI-PLANT PRODUCTION: ALGEBRAIC EXAMPLE (no price discrimination)

In the stylized world of algebra, we assume that each plant can produce little fractions of units of output; we also assume that the firm can sell little fractions of output. Perhaps you can see that under these assumptions, the firm should increase production at each plant as long as the marginal cost at the plant is even slightly lower than the marginal revenue from the sale of the product; this ensures that the firm captures all of the extra profit possible—even tiny fractions of profit as marginal cost approaches marginal revenue. In the limit, this means:

Profit-maximizing rule for a two plant firm:

$$MC_1 = MC_2 = MR$$

Graphically, the rule looks like this: (scale is off)



Take some time to study this graph and make sure that you understand the concepts presented thus far. You need to fully understand the concepts in order to grasp the algebraic example that follows.

Here's an example:

A monopoly has demand curve $P = 100 – Q$; hence it has $MR = 100 – 2Q$

It has two plants.
Plant 1 has MC curve $MC_1 = .5Q_1$
Plant 2 has MC curve $MC_2 = .25Q_2$

(Note that $Q_1 + Q_2 = Q$; that is, the sum of production at both plants is sold to buyers)

Price discrimination is illegal. What is the profit-maximizing strategy?

Solution: There are a number of ways to solve this problem. Here is perhaps the easiest.

Step 1: (Here, we are "horizontally summing" the two MC curves)
a) Rearrange the MC curves to get the Q's on the left hand side
b) Add the two rearranged MC curves together to get a sum
c) Rearrange the new summed MC curve to get MC back on the left

Step 2: Set MR = the MC curve found in step 1, part (c).
a) Solve for Q
b) Solve for MC

Step 3. Substitute MC found in step 2, part (b) into MC_1 and MC_2 to find Q_1 and Q_2
Check to make sure that $Q_1 + Q_2 = Q$

Step 1:

a) $MC_1 = .5Q_1 \rightarrow Q_1 = 2MC_1$

$MC_2 = .25Q_2 \rightarrow Q_2 = 4MC_2$

b) Sum: $(Q_1+Q_2) = 6MC \rightarrow Q = 6MC_{summed}$

c) Rearrange: $MC_{summed} = (1/6)Q$

Step 2:

a) $MR = MC_{summed} \rightarrow 100 - 2Q = (1/6)Q \rightarrow 2.1667Q = 100 \rightarrow Q = 46.153$

b) $MC = (1/6)Q = (1/6)46.1538 = 7.6923$

Step 3:

$Q_1 = 2MC_1 \rightarrow Q_1 = 2(7.5923) = 15.384$
$Q_2 = 4MC_1 \rightarrow Q_2 = 4(7.5923) = 30.769$

Check: $Q_1 + Q_2 = Q$?

15.384 + 30.769 = 46.153 Yes, thank goodness.

Hence the firm will produce 15.384 units at plant 1 and 30.769 units at plant 2, selling the combined production—46.153 units—to buyers. How much will it charge for each unit? We can plug Q = 46.153 into the demand curve to get this answer:

$P = 100 - 46.153 = 53.847$

What is the firm's total revenue?

total revenue = P x Q = \$53.847 x 46.153 = \$2485.20