Investment, Time, and Present Value

Contents:

Introduction

Decisions made by a firm manager today often result in cash flows which occur not only today but also years into the future. For example, a manufacturer may purchase a machine today—an immediate cost—which results in higher revenues for the firm for years to come, since the machine enhances the firm's production process.

It is a bit tricky to evaluate the merits of a decision whose costs and benefits occur over time. For example, suppose I offered you an investment option—invest $100 and receive a return of $200. Is this a good investment? The answer depends when you get the return of $200. If you get it next year, then it's good. If you get it in 75 years then it ain't so good.

So how does one precisely measure costs and benefits of actions which have financial consequences over time? The concept of **present value** is very useful. We shall discuss present value presently. But first, let's look at a closely related topic that tends to be more intuitive—*future value*.

Future value

Suppose you are offered a certain financial investment—place $100 in the investment today, and receive a 5% annual rate of return (simple interest[1]), for as long as you want.

What will your investment's value be in 1 year?

Future value of $100 in 1 year, 5% annual return = 100 x 1.05 = $105

How about 2 years? (In the second year, the $105 will earn 5% interest.)

Future value of $100 in 2 years, 5% annual return = 105 x 1.05 = $110.25

How about 3 years? (In the third year, the $110.25 will earn 5% interest.)

Future value of $100 in 3 years, 5% annual return = 110.25 x 1.05 = $115.7625

[1] Throughout our analysis we shall assume simple interest—the equivalent of compounding interest annually. In corporate finance classes, you will encounter other types of interest, e.g. compounded monthly, continuously, etc. We also ignore taxes.

Oh, if only there were a formula that could calculate the future value of a sum of money. Hey, there is such a formula! It's on the next page of notes!

$$\mathbf{FV = PV(1 + r)^n}$$

"FV" is future value
"PV" is present value—the value of the money today
r is the rate of return
n is the number of years in the future

For example, how much will your $100 investment be worth in 21 years?

$FV = 100(1 + .05)^{21} = \$278.5963$

(There are more complicated versions of this formula, for investments with uncertain rates of return, and ones with compounding occurring more than once a year, etc. You will encounter these more complicated formulas in corporate finance classes.)

We have seen how to calculate the future value of $ invested today. But can we calculate the value today—the present value—of a future sum? Yes!

<u>Present Value</u>
Suppose some dude offered you a certain investment—he will certainly give you $1000 ten years from now. How much are you willing to pay him now to receive a certain $1000 in ten years? You will pay less than $1000, since you can invest the money yourself in a Treasury bond that pays 6% rate of return (and let's assume that this is a certain payment).

We can use the future value equation to calculate the maximum amount of money that you would be willing to pay the dude today to get $1000 with certainty in the future:

$$FV = PV(1 + r)^n$$

Divide both sides by $(1 + r)^n$

$$\frac{FV}{(1+r)^n} = PV$$

Now, for convention's sake, switch sides:

$PV = \frac{FV}{(1+r)^n}$	←Hey! This is the present value equation!!!!

Note that we divide FV by a number greater than 1, that depends on r – the rate of return. Hence PV is less than FV. (Hopefully this makes sense to you.) We are said to *discount* the future value; hence r is called the *discount rate*.

Let's use the present value equation to calculate the most that you would pay the dude today for his investment. Assume that you could receive a 6% rate of return with certainty elsewhere in the economy from other investments.

$$PV = \frac{1000}{(1+.06)^{10}} = \$558.3948$$

The most that you'd be willing to pay NOW to get $1000 in 10 years is $558.3948. If you can pay less than that today and get $1000 in 10 years then the investment is better than the Treasury bond option (at a 6% return). If you must pay more, than the Treasury bond option is superior.

<u>The present value of a stream of cash flows</u>
Suppose that you won the lotto, paying you $1,000,000 per year for 25 years. Now some dudette offers to buy your winning lotto ticket for $15 million. Should you accept the offer? (And let's ignore taxes here, as we have done throughout these notes.)

Well, we can use the present value formula 25 times, to calculate the present value of each of the $1,000,000 payments. Then we can add up the 25 results, to get the present value of this stream of twenty-five $1 million cash flows. (We should accept the dudette's $15 million offer if it is greater than the present value of the alternative—the 25 payments.)

A difficulty arises: what do we use for r – the discount rate? We should use the rate of return that we would expect if we accept the dudette's buyout offer and invest the $15 million in some financial instrument. This return is uncertain and is crucial in the calculation. For now, let's just assume an 8% return.

For example:

PV of the first $1 million $PV = \frac{1000}{(1+.08)^{0}}$ = $1 million

PV of the second $1 million $PV = \frac{1000}{(1+.08)^{1}}$ = $925,926

In the handy table on the next page, I present the PVs of all the 25 payments, at 8% discount rate.

year	payment	pv of payment
0	1,000,000.00	1,000,000.00
1	1,000,000.00	925,925.93
2	1,000,000.00	857,338.82
3	1,000,000.00	793,832.24
4	1,000,000.00	735,029.85
5	1,000,000.00	680,583.20
6	1,000,000.00	630,169.63
7	1,000,000.00	583,490.40
8	1,000,000.00	540,268.88
9	1,000,000.00	500,248.97
10	1,000,000.00	463,193.49
11	1,000,000.00	428,882.86
12	1,000,000.00	397,113.76
13	1,000,000.00	367,697.92
14	1,000,000.00	340,461.04
15	1,000,000.00	315,241.70
16	1,000,000.00	291,890.47
17	1,000,000.00	270,268.95
18	1,000,000.00	250,249.03
19	1,000,000.00	231,712.06
20	1,000,000.00	214,548.21
21	1,000,000.00	198,655.75
22	1,000,000.00	183,940.51
23	1,000,000.00	170,315.28
24	1,000,000.00	157,699.34
		11,528,758.28

The dudette's offer of $15 million looks awfully good, compared to the PV of the 25 payments--$11.5 million! (In other words, if you take the dudette's offer of $15 million and you earn an 8% return with it then you will do much better than keeping the lotto ticket—around $3.5 million better in today's dollars.)

(By the way, some financial calculators can calculate the present value of a stream of equal cash flows using only a few commands. Consult your calculator's manual.)

Using Present Value to Analyze Investment Decisions: Net Present Value

Now suppose that you are a firm thinking of making an investment; it will cost money today, and will result in higher revenue in the future. There will be a cash outflow today, and cash inflows in the future.

You would like to evaluate the merits of this investment, relative to alternate investments that you could make. The *net present value* of the investment is the present value of all of the cash outflows and inflows that result from the investment.

> If an investment's net present value is positive, then it is a superior investment relative to the average investment in the economy. The higher an investment's net present value, the more superior it is relative to the average investment.

Example:
A machine costs $1000 to buy. It will result in $300 of revenue at the end of each year, for 4 years. At the end of the four years it can be sold for scrap for $100.

This investment has five cash flows:

Now:	-$1000 (the cost of the machine)
1 year from now:	$300
2 years from now:	$300
3 years from now:	$300
4 years from now:	$300 + $100 = $400 (revenue + scrap value)

Let us use the present value formula to calculate the PV of each of these cash flows. Then we'll add up the PVs to get the Net Present Value (NPV).

One important question: what discount rate to use? Answer: the discount rate should equal the rate of return that an average investment with similar risk would enjoy. (Later in these notes, we shall discuss how one might calculate this rate of return.) For now, let's just say that it is 11%.

$$\text{PV of cash flows} = \frac{-1000}{1.11^0} + \frac{300}{1.11^1} + \frac{300}{1.11^2} + \frac{300}{1.11^3} + \frac{400}{1.11^4} = \quad -\$3.39$$

The negative NPV tells us that we could do better with an average investment elsewhere in the economy, so this one doesn't look very good.

The appropriate discount rate, and the Capital Asset Pricing Model (CAPM)

What discount rate to use? We have previously written that the discount rate used to evaluate an investment should be the rate of return that one would expect on the average investment with similar risk. Let us digress briefly on the relationship between risk and return.

Risk and return:
Usually, an investment in which the returns are less likely to occur (more risky) must offer a higher rate of return. For example, when Planet Hollywood, in its inception, needed to raise money, they issued bonds. Potential investors saw the Planet Hollywood venture as very risky, so they demanded a high return on the bonds. (The idea is: why invest in Planet Hollywood bonds, rather than Treasury bonds, which are virtually risk free? The only reason would be if the Planet Hollywood bonds paid a higher expected return, since there is a much higher *default risk* on Planet Hollywood bonds—a higher likelihood that they will not meet their bond obligations.)

Suppose we have an investment—call it investment "i"—and we want to know what kind of rate of return that it will command. The rate of return—call it r_i—depends in part on how risky we expect that the investment is. Suppose we can express the riskiness of investment i with a number—call it "β" or "beta."

CAPM:
The Capital Asset Pricing Model is a theoretical relationship between the risk of an investment and its expected return. CAPM can be summarized in one equation:

$$\mathbf{r_i = r_f + \beta_i(r_m - r_f)}$$ ← the capital asset pricing model

what are these symbols?

$\mathbf{r_i}$	is the expected rate of return on investment i
$\mathbf{r_f}$	is the rate of return on a risk-free investment
$\mathbf{r_m}$	is the average rate of return of all investments in the economy
$\boldsymbol{\beta_i}$	"beta," it is the riskiness of the return of investment i

Let's comment further on these symbols

$\boldsymbol{\beta_i}$
This is investment i's predicted riskiness. It will have a value greater than or equal to zero. If it equals zero, then investment i is a risk free investment, and it will pay its expected return with certainty. If it equals one, then investment i has the same risk as the average investment in the economy.

(How does one calculate beta? It is often quite difficult to do this, especially for a new investment. One tries to find a similar investment that has been made in the past, and use statistical techniques to calculate its beta; then this beta is applied to the new investment. You will explore this topic further in corporate finance or investment class.)

r_i
This is the rate of return that we would expect, on average, from an investment that has riskiness level of β_i

r_m
If one averaged the rates of return of all investments, one would, in theory, get r_m. It is impossible, really, to do this kind of average, so in the real world r_m is approximated by using the average rate of return of some mutual fund that holds a lot of stocks (such as an S&P 500 mutual fund).

r_f
This is the return that we would receive from a theoretical investment with zero risk—an investment whose returns are completely guaranteed. There is no such investment in the real world; often the return on Treasury bonds is used for r_f, since this return is the closest thing to being guaranteed.

<u>CAPM and the discount rate</u>
If you are a firm thinking about making an investment—say, investment i—and you can estimate its risk—its beta—then you know the discount rate to apply to the cash flows that you expect from the investment; you should use the r_i calculated from the CAPM equation as the discount rate.

Example:
You are considering buying a robot for $100 which will result in revenues of $50 at the end of the year for 2 years. Then you can sell it for scrap for $20. You have calculated the beta of this investment at beta=1.1. Treasury bonds pay 6% return. An S&P 500 mutual fund pays an average 8% return.

Is your robot a worthwhile investment?

Answer: first, let's calculate r_i using the CAPM equation

$$r_i = r_f + \beta_i(r_m - r_f)$$

$$r_i = .06 + 1.1(.08 - .06)$$

$$r_i = .082$$

This r_i is the discount rate that we should use. Now let's figure out the NPV of the robot investment.

$$\text{NPV} = \frac{-100}{1.082^0} + \frac{50}{1.082^1} + \frac{70}{1.082^2} = \$6.00278$$

This positive NPV indicates that the robot investment is a good one—better than the average investment.