

Study Question – 2 Plants

A firm has total revenue equation and 2 plants $TR = 5000Q - 5Q^2$

total cost equation, plant A $C_a = 500 + 3Q_a^2$

total cost equation, plant B $C_b = 100 + 2Q_b^2$

where $Q = Q_a + Q_b$

Calculate this firm's profit maximizing price, total quantity, quantity produced at each plant, total revenue, total cost, and total profit. (This firm does not price discriminate.)

Marginal revenue is the derivative of the total revenue equation with respect to Q:

$$MR = 5000 - 10Q$$

Marginal cost at plant A is the derivative of total cost at plant A:

$$MC_a = 6Q_a$$

$$MC_b = 4Q_b$$

Rearrange MC_a and MC_b to get Q_a and Q_b by themselves on one side of the equation:

$$Q_a = (1/6)MC_a$$

$$Q_b = (1/4)MC_b$$

Sum the two equations

$$(Q_a + Q_b) = (5/12)MC_{\text{summed}}$$

$$Q = (5/12)MC_{\text{summed}}$$

Rearrange the last equation to get MC_{summed} by itself on one side of the equation:

$$MC_{\text{summed}} = 2.4Q$$

Set $MR = MC_{\text{summed}}$:

$$5000 - 10Q = 2.4Q$$

Solve for Q:

$$12.4Q = 5000 \rightarrow$$

$$Q = 403.2258$$

Plug $Q = 403.2258$ into MR equation:

$$MR = 5000 - 10(403.2258) = \$967.7419$$

We know that $MR = MC_a = MC_b$, so use 967.7419 for MC_a and MC_b to solve for Q_a and Q_b :

$$Q_a = (1/6)MC_a = (1/6)(967.7419) = 161.2903 \text{ units}$$

$$Q_b = (1/4)MC_b = (1/4)(967.7419) = 241.9355 \text{ units}$$

To get total revenue, substitute $Q = 403.2258$ into total revenue equation:

$$TR = 5000(403.2258) - 5(403.2258^2) = 1203174$$

To get price per unit (i.e. average revenue), divide total revenue by the number of units, 403.2258

$$P = TR/Q = 1203174/403.2258 = \$2983.871$$

To get total cost at plant A, plug in $Q_a = 161.2903$ into plant A's total cost equation:

$$C_a = 500 + 3(161.2903^2) = 78543.68$$

To get total cost at plant B, plug $Q_b = 241.9355$ into plant B's total cost equation:

$$C_b = 100 + 2(241.9355^2) = 117165.6$$

Total cost at the firm is the sum of the costs at plant A and plant B:

$$\text{Total cost} = 78543.68 + 117165.6 = 195709.3$$

Total profit equals total revenue minus total cost:

$$\text{Total profit} = 1203174 - 195709.3 = 1007465$$