

Study Question -- Duopoly

In a duopoly there exist firms each with constant average costs of \$4. Demand for the combined output of the firms is expressed with the demand equation $Q = 100 - .5P$.

Calculate the following:

- a) Cartel equilibrium price and quantity
- b) Cournot equilibrium price and quantity
- c) Stackelberg equilibrium price and quantity
- d) Bertrand equilibrium price and quantity

Rearrange demand equation: $P = 200 - 2Q$

Note also that since average cost is constant at \$4, $MC = \$4$

a) $MR = 200 - 4Q$

$$MR = MC \rightarrow 200 - 4Q = 4 \rightarrow Q = 49 \rightarrow P = 200 - 2(49) = \$102$$

(Q must be split between firms A and B; we have insufficient information to determine the precise split.)

b) total revenue firm a = $PQ_a = (200 - 2Q)Q_a = 200Q_a - [2(Q_a + Q_b)]Q_a$

$$= 200Q_a - (2Q_a + 2Q_b)Q_a = 200Q_a - 2Q_a^2 - 2Q_bQ_a$$

marginal revenue firm a = derivative of total revenue with respect to Q_a

$$MR_a = 200 - 4Q_a - 2Q_b$$

Set $MR_a = MC_a$ to get firm a's reaction equation:

$$200 - 4Q_a - 2Q_b = 4 \rightarrow 196 - 2Q_b = 4Q_a \rightarrow 49 - .5Q_b = Q_a$$

By symmetry, firm b's reaction equation is $49 - .5Q_a = Q_b$

Substitute firm b's reaction equation into firm a's reaction equation:

$$49 - .5(49 - .5Q_a) = Q_a \rightarrow 49 - 24.5 + .25Q_a = Q_a$$

$$\rightarrow .75Q_a = 24.5 \rightarrow Q_a = 32.666666667 \text{ (rounded)}$$

By symmetry, $Q_b = 32.666666667 \text{ (rounded)}$

$$Q = Q_a + Q_b = 65.3333333333 \text{ (rounded)}$$

$$P = 200 - 2(65.333333) = 69.3333333333$$

c) Substitute firm b's reaction equation into firm a's total revenue equation:

$$R_a = 200Q_a - 2Q_a^2 - 2(49 - .5Q_a)Q_a$$

$$R_a = 200Q_a - 2Q_a^2 - (98 + Q_a)Q_a$$

$$R_a = 200Q_a - 2Q_a^2 - 98Q_a - Q_a^2$$

$$R_a = 102Q_a - Q_a^2$$

marginal revenue firm a = derivative of total revenue with respect to Q_a

$$MR_a = 102 - 2Q_a$$

Set $MR_a = MC_a$:

$$102 - 2Q_a = 4 \rightarrow Q_a = 49$$

Plug $Q_a = 49$ into firm b's reaction equation:

$$Q_b = 49 - .5(49) = 24.5$$

$$Q = Q_a + Q_b = 49 + 24.5 = 73.5 \rightarrow P = 200 - 2(73.5) = \$53$$

d) price is driven down to marginal cost : $P = \$4 \rightarrow Q = 98$